Geometry, Structure and Randomness in Combinatorics

edited by

Jiří Matoušek, Jaroslav Nešetřil and Marco Pellegrini

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Preface

On September 3-7, 2012, as part of the activities of the Mathematics Research Center "Ennio De Giorgi" and on the invitation of its director prof. Mariano Giaquinta, we organized the Workshop "Geometry, Structure and Randomness in Combinatorics" at Scuola Normale Superiore in Pisa. The workshop was organized by Jiří Matousek, Jaroslav Nešetřil (Charles University, Prague) and Marco Pellegrini (CNR, Pisa) and has been supported jointly by SNS and CRM Pisa and DIMATIA centre in Prague.

This workshop intended to reflect some key recent advances in combinatorics, particularly in the area of extremal theory and Ramsey theory. It also aimed to demonstrate the broad spectrum of techniques and its relationship to other fields of mathematics, particularly to geometry, logic and number theory.

Invited speakers included ten of the leading experts. We had the pleasure to invite Prof. Endre Szemerédi, the winner of the Abel Prize in 2012 for his fundamental contributions in the field of discrete mathematics and theoretical computer science. The workshop attracted 48 participants both from Italy and abroad.

The following list is that of the invited lectures at the workshop:

IMRE BARANY, *Tensors, colours, and octahedral*

BÉLA BOLLOBÁS, *Extremal and probabilistic results on bootstrap percolation*

MARIA CHUDNOVSKY, *Extending the Gyarfas-Sumner conjecture*

ZEEV DVIR, *Configurations of points with many collinear triples: going beyond Sylvester-Gallai*

ZOLTAN FUREDI, *Binary codes versus hypergraphs*

JAROSLAV NEŠETŘIL, *A unifying approach to graph limits II*

PATRICE OSSONA DE MENDEZ, *A unifying approach to graph limits I*

ALEX SCOTT, *Discrepancy in graphs, hypergraphs and tournaments*
and (second talk)
Szemerédi regularity lemma for sparse graphs

JOZSEF SOLYMOSI, *Sums vs. products*
and (second talk)
The (7,4)*-conjecture for finite groups*

ENDRE SZEMERÉDI, *On subset sums*

Given the success of both scientific and public workshops, at the end of the event, at the suggestion of Professor Mariano Giaquinta, it has been proposed to organize a volume dedicated to this meeting. This proposal was welcomed by all the speakers. The present volume has been edited for the "CRM Series", with the title "Geometry, Structure and Randomness in Combinatorics" and includes both original scientific articles in extended form or survey articles on results and problems inherent in the themes presented at the workshop. Each article submitted was reviewed.

We thank all the authors for their contribution and again Scuola Normale Superiore and its Centro di Ricerca Matematica Ennio De Giorgi and to DIMATIA Centre of Charles University for their generous support.

Pisa/Prague

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Tensors, colours, octahedra

Imre Bárány

Abstract. Several theorems in combinatorial convexity admit colourful versions. This survey describes old and new applications of two methods that can give such colourful results. One is the octahedral construction, the other is Sarkaria's tensor method.

1 Introduction

Theorems of Carathéodory, Helly, and Tverberg are classical results in combinatorial convexity. They all have coloured versions. Some others involve colours directly. For instance in Kirchberger's theorem [15], the elements of a finite set $X \subset \mathbb{R}^d$ are coloured Red and Blue, and the statement is that the Red and Blue points can be separated by a hyperplane if and only if for every $Y \subset X$ with $|Y| \leq d + 2$, the Red and Blue points in Y can be separated by a hyperplane.

The aim of this paper is to describe and explain old and new applications of two methods that have turned out to be useful when proving such colourful results. One is the octahedral construction, discovered and first used by László Lovász in 1991, which appeared in [4]. The other is Karinbir Sarkaria's tensor method, originally from [25] and developed further in [5].

In the next section Tverberg's theorem and its colourful version are presented. The octahedral construction is given in Section 3 with applications followed in later sections.

2 Tverberg's theorem and its coloured version

Tverberg's theorem is a gem, one of my favourites. Here is what it says.

Theorem 2.1. *Assume $d \geq 1, r \geq 2$ and $X \subset \mathbb{R}^d$ has $(r-1)(d+1)+1$ elements. Then X has a partition into r parts $X_1, \ldots, X_r$ such that $\bigcap_1^r \operatorname{conv} X_i \neq \emptyset$.*