Asymptotics in Dynamics, Geometry and PDEs; Generalized Borel Summation vol. II

edited by
O. Costin, F. Fauvet,
F. Menous, D. Sauzin

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Introduction

In the last three decades or so, important questions in distinct areas of mathematics such as the local analytic dynamics, the study of analytic partial differential equations, the classification of geometric structures (*e.g.* moduli for holomorphic foliations), or the semi-classical analysis of Schrödinger equation have necessitated in a crucial way to handle delicate asymptotics, with the formal series involved being generally divergent, displaying specific growth patterns for their coefficients, namely of Gevrey type. The modern study of Gevrey asymptotics, for questions originating in geometry or analysis, goes together with an investigation of rich underlying algebraic concepts, revealed by the application of Borel resummation techniques.

Specifically, the study of the Stokes phenomenon has had spectacular recent applications in questions of integrability, in dynamics and PDEs. Some generalized form of Borel summation has been developed to handle the relevant structured expansions – named transseries – which mix series, exponentials and logarithms; these formal objects are in fact ubiquitous in special function theory since the 19th century.

Perturbative Quantum Field Theory is also a domain where recent advances have been obtained, for series and transseries which are of a totally different origin from the ones met in local dynamics and yet display the same sort of phenomena with, strikingly, the very same underlying algebraic objects.

Hopf algebras, *e.g.* with occurrences of shuffle and quasishuffle products that are important themes in the algebraic combinatorics community, appear now natural and useful in local dynamics as well as in pQFT. One common thread in many of the important advances for these questions is the concept of resurgence, which has triggered substantial progress in various areas in the near past.

An international conference took place on October 12th – October 16th, 2009, in the Centro di Ricerca Matematica Ennio De Giorgi, in Pisa, to highlight recent achievements along these ideas.

Here is a complete list of the lectures delivered during this event:

Carl Bender, *Complex dynamical systems*

Filippo Bracci, *One resonant biholomorphisms and applications to quasi-parabolic germs*

David Broadhurst, *Multiple zeta values in quantum field theory*

Jean Ecalle, *Four recent advances in resummation and resurgence theory*

Adam Epstein, *Limits of quadratic rational maps with degenerate parabolic fixed points of multiplier* $e^{2\pi i q} \to 1$

Gérard Iooss, *On the existence of quasipattern solutions of the Swift-Hohenberg equation and of the Rayleigh-Benard convection problem*

Shingo Kamimoto, *On a Schrödinger operator with a merging pair of a simple pole and a simple turning point, I: WKB theoretic transformation to the canonical form*

Tatsuya Koike, *On a Schrödinger operator with a merging pair of a simple pole and a simple turning point, II: Computation of Voros coefficients and its consequence*

Dirk Kreimer, *An analysis of Dyson Schwinger equations using Hopf algebras*

Joel Lebowitz, *Time Asymptotic Behavior of Schrödinger Equation of Model Atomic Systems with Periodic Forcings: To Ionize or Not*

Carlos Matheus, *Multilinear estimates for the 2D and 3D Zakharov-Rubenchik systems*

Emmanuel Paul, *Moduli space of foliations and curves defined by a generic function*

Jasmin Raissy *Torus actions in the normalization problem*

Javier Ribon, *Multi-summability of unfoldings of tangent to the identity diffeomorphisms*

Reinhard Schäfke, *An analytic proof of parametric resurgence for some second order linear equations*

Mitsuhiro Shishikura, *Invariant sets for irrationally indifferent fixed points of holomorphic functions*

Harris J. Silverstone, *Kramers-Langer-modified radial JWKB equations and Borel summability*

Yoshitsugu Takei, *On the turning point problem for instanton-type solutions of (higher order) Painlevé equations*

Saleh Tanveer, *Borel Summability methods applied to PDE initial value problems*

Jean-Yves Thibon, *Noncommutative symmetric functions and combinatorial Hopf algebras*

Valerio Toledano Laredo, *Stokes factors and multilogarithms*

Stefan Weinzierl, *Feynman graphs in perturbative quantum field theory*
Sergei Yakovenko, *Deflicity of intersections between trajectories of ODEs and algebraic hypersurfaces*
Michael Yampolsky, *Geometric properties of a parabolic renormalization fixed point*

The present volume, together with a first one already published in the same collection, contains five contributions of invited speakers at this conference, reflecting some of the leading themes outlined above.

We express our deep gratitude to the staffs of the Scuola Normale Superiore di Pisa and of the CRM Ennio de Giorgi, in particular to the Director of the CRM, Professor Mariano Giaquinta, for their dedicated support in the preparation of this meeting; all participants could thus benefit of the wonderful and stimulating atmosphere in these institutions and around Piazza dei Cavalieri. We are also very grateful for the possibility to publish these two volumes in the CRM series. We acknowledge with thankfulness the support of the CRM, of the ANR project "Resonances", of Université Paris 11 and of the Gruppo di Ricerca Europeo Franco-Italiano: Fisica e Matematica, with also many thanks to Professor Jean-Pierre Ramis. All our recognition for the members of the Scientific Board for the conference: Professors Louis Boutet de Monvel (Univ. Paris 6), Dominique Cerveau (Univ. Rennes), Takahiro Kawai (RIMS, Kyoto) and Stefano Marmi (SNS Pisa).

Pisa, June 2011

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