

Contents

1	General Discussion of Lie Algebras	1
1.1	Introduction	1
1.2	Definition of a Lie Algebra	1
1.2.1	Example: the general linear Lie algebra	3
1.3	Representations of a Lie Algebra	4
1.3.1	Examples of matrix representations	4
1.4	Realizations of a Lie Algebra	5
1.4.1	Examples of realizations	7
1.5	Irreducible Representations	10
1.5.1	Example: a reducible representation	11
1.6	Exercises	12
2	Commutator Gymnastics	17
2.1	Introduction	17
2.2	General Commutator Identities	17
2.3	Commutators Involving r and p	19
2.4	Commutators Involving L	21
2.5	Exercises	23
3	Angular Momentum Theory and $so(3)$	25
3.1	Introduction	25
3.2	Ladder Operators for $so(3)$	26
3.3	General Eigenvalue Spectrum of J_3	28
3.4	Unirreps of $so(3)$	28
3.5	$so(3)$ Scalar Operators	30
3.6	$so(3)$ Vector Operators	30
3.7	Selection Rules for a Vector Operator	31
3.8	Matrix Elements of a Vector Operator	33
3.9	Wigner-Eckart Theorem	37
3.10	Exercises	39
4	Representations and Realizations of $so(2,1)$	41
4.1	Introduction	41
4.2	Ladder Operators for $so(2,1)$	42
4.3	Unirreps of $so(2,1)$	44

4.4	Realizations of $so(2,1)$	45
4.5	3-dimensional Hydrogenic Atom	50
4.6	3-dimensional Harmonic Oscillator	53
4.7	Generalization to D Dimensions	54
4.8	D-dimensional Hydrogenic Atom	56
4.9	D-dimensional Harmonic Oscillator	56
4.10	Hermiticity of $so(2,1)$ Generators	58
4.11	Explicit T_3 Eigenfunctions	60
4.12	D-dimensional Relativistic Systems	66
4.12.1	Klein-Gordon equation	66
4.12.2	Dirac-Coulomb equation	68
4.13	General Radial Equations	71
4.14	Parabolic Hydrogenic Realization of $so(2,1)$	72
4.14.1	Parabolic coordinate system	72
4.14.2	Separation in parabolic coordinates	73
4.14.3	Algebraic solution: method 1	76
4.14.4	Algebraic solution: method 2	77
4.15	Exercises	78
5	Representations and Realizations of $so(4)$	81
5.1	Introduction	81
5.2	Representations of $so(4)$	83
5.3	$so(4)$ as a Coupling Problem	89
5.4	Laplace-Runge-Lenz Vector	94
5.4.1	Classical case	94
5.4.2	Quantum mechanical case	95
5.5	Hydrogenic Realizations of $so(4)$	97
5.5.1	Pauli's hydrogenic realization of $so(4)$	98
5.5.2	Scaled hydrogenic realization of $so(4)$	100
5.6	Exercises	102
6	Scaled Hydrogenic Realization of $so(4,2)$	105
6.1	Introduction	105
6.2	Scaled Hydrogenic Realization of $so(4,1)$	106
6.3	Scaled Hydrogenic Realization of $so(4,2)$	108
6.4	Matrix Representation of $so(4,2)$	111
6.5	Hydrogenic Tower of States	114
6.6	Exercises	116
7	Lie Algebraic Perturbation Theory	119
7.1	Introduction	119
7.2	Conventional RSPT	120
7.2.1	Conventional iteration formula	122
7.2.2	Conventional low order formulas	122
7.2.3	Conventional high-order iteration scheme	123

7.2.4	Example: continuum difficulties for Stark effect	124
7.2.5	Dalgarno and Lewis method for Stark effect	125
7.3	Scaled Hydrogenic Hamiltonian	127
7.4	Modified Algebraic RSPT	129
7.4.1	Modified iteration formula	131
7.4.2	Symmetric energy formula	131
7.4.3	Modified low order formulas	132
7.4.4	Modified high order iteration scheme	132
7.4.5	Example: ground state Stark effect	134
7.5	Exercises	136
8	Symbolic Calculation of the Stark Effect	137
8.1	Introduction	137
8.2	Symbolic Calculations for the Ground State	138
8.2.1	Modified high order formalism	138
8.2.2	Renormalized matrix elements	141
8.2.3	Program <i>gsstark</i> for $E^{(j)}$ and $\Psi^{(j)}$	144
8.2.4	Program for symmetric energy formula	149
8.3	Summing the Ground State Series	152
8.3.1	Direct summation of the series	152
8.3.2	Summation using Padé approximants	153
8.4	Stark Effect in Parabolic Coordinates	158
8.4.1	Separation and perturbation formalism	158
8.4.2	Maple program for $f^{(0)}$ to $f^{(4)}$	162
8.4.3	Unravelling the energy expansion	164
8.4.4	Higher orders: general case	166
8.4.5	Higher orders: special cases	172
8.5	2-Dim Stark Effect in Parabolic Coordinates	174
8.6	Exercises	177
9	Symbolic Calculation of the Zeeman Effect	179
9.1	Introduction	179
9.2	Low Order Calculations for the Ground State	180
9.3	Symbolic Calculations for the Ground State	182
9.3.1	Modified high order formalism	182
9.3.2	Renormalized matrix elements	184
9.3.3	Program <i>gszeeman</i> for $E^{(j)}$ and $\Psi^{(j)}$	186
9.3.4	Program for symmetric formula	189
9.3.5	Summation of the ground state series	191
9.4	Degenerate RSPT for the Zeeman Effect	191
9.4.1	Doubly-degenerate modified RSPT	192
9.4.2	The $3s-3d_0$ degenerate level	195
9.4.3	Procedure <i>zee3s3d</i> for the $3s$ and $3d_0$ states	197
9.4.4	Energy corrections for the $3s$ and $3d_0$ states	204
9.5	Method of Moments	206

9.6 Exercises	209
10 Spherically Symmetric Systems	211
10.1 Introduction	211
10.2 RSPT for Charmonium and Harmonium	212
10.2.1 Low order calculations	213
10.2.2 Symbolic calculations	214
10.2.3 Maple programs for charmonium and harmonium	216
10.3 RSPT for Screened Coulomb Potential	226
10.4 Power Series Method	228
10.5 HVHF Perturbation Method	233
10.5.1 Hypervirial and Hellmann-Feynman theorems	234
10.5.2 HVHF method for hydrogenic systems	234
10.6 HVHF Method for Power Potentials	236
10.7 HVHF Method: Screened Coulomb Case	240
10.8 Exercises	243
A The Levi-Civita Symbol	245
A.1 Basic Properties	245
A.2 Important Identities	245
A.3 Exercises	246
B Lie Groups and Lie Algebras	247
B.1 Some Definitions	247
B.2 One-parameter Subgroups	248
B.3 Rotation Group $SO(2)$	250
B.4 Rotation Group $SO(3)$	250
B.5 Special Unitary Group $SU(2)$	252
B.6 Mapping from $SU(2)$ to $SO(3)$	253
B.7 $SO(n)$ and its Lie Algebra $so(n)$	253
B.7.1 Special case: $so(3)$	255
B.7.2 Special case: $so(4)$	256
B.8 $SO(p,q)$ and its Lie Algebra $so(p,q)$	256
B.8.1 Special case: $so(2,1)$	258
B.8.2 Special case: $so(4,1)$	259
B.8.3 Special case: $so(4,2)$	259
B.9 Exercises	260
C The Tilting Transformation	261
C.1 Transformation of Functions	261
C.2 Transformation of Operators	262
C.3 Transformation of the Radial Equation	262
C.3.1 General case	262
C.3.2 Special case of hydrogenic atom	263
C.4 Tilted Laplace-Runge-Lenz Vector	264

C.5 Exercises	264
D Perturbation Matrix Elements	265
D.1 Basic Formulas for Matrix Elements	265
D.1.1 3-dimensional case	265
D.1.2 D-dimensional case	266
D.2 Symbolic Calculation of Tables	267
D.3 Special Values of Matrix Elements	274
D.3.1 Matrix elements of R for $n = 1, \dots, 5$	274
D.3.2 Matrix elements of $W = RZ$ for $n = 1, \dots, 5, m = 0$	275
E Tables of Stark Effect Energy Corrections	281
E.1 Ground State Energy to Order 100	281
E.2 Symbolic Energy Corrections for Parabolic States	288
F Tables of Zeeman Effect Energy Corrections	295
F.1 Ground State Energy to Order 60	295
G Tables of Charmonium Energy Corrections	313
G.1 Ground State Energy to Order 100	313
G.2 Symbolic Energy Corrections to Order 14	326
H Tables of Harmonium Energy Corrections	331
H.1 Ground State Energy to Order 100	331
H.2 Symbolic Energy Corrections to Order 10	349
I Tables of Screened Coulomb Energy Corrections	355
I.1 Ground State Energy to Order 100	355
I.2 Ground State Yukawa Energy for D Dimensions	368
I.3 Energy Series for Screened Coulomb Potential	371
I.4 Energy Series for Yukawa Potential	375
J Solutions to Exercises	379
J.1 Solutions to Chapter 1 Exercises	379
J.2 Solutions to Chapter 2 Exercises	385
J.3 Solutions to Chapter 3 Exercises	389
J.4 Solutions to Chapter 4 Exercises	393
J.5 Solutions to Chapter 5 Exercises	399
J.6 Solutions to Chapter 6 Exercises	409
J.7 Solutions to Chapter 7 Exercises	415
J.8 Solutions to Chapter 8 Exercises	424
J.9 Solutions to Chapter 9 Exercises	424
J.10 Solutions to Chapter 10 Exercises	426
J.11 Solutions to Appendix A Exercises	427
J.12 Solutions to Appendix B Exercises	428
J.13 Solutions to Appendix C Exercises	432

Bibliography	435
Index of Symbols Used	443
Index	447