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Basics of Superconductivity

An Introduction to Theoretical Physics

tredition

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Preface

Aim of the present textbook is to give a thorough introduction to important theories of superconductivity in such a way that the reader will be able to get a deep understanding of the prominent effects in superconductors. Particular importance is attached to explain the basic properties of superconductivity and its most important applications. Thereby, in the present textbook we focus on the phenomenological theories of superconductivity. This means that the theoretical concepts are based on a few assumptions about the superconducting state, which are motivated by experiments, without going into the details of the underlying microscopic system. This is completely sufficient for a good understanding of the phenomena as this book strives for. The treatment of the microscopic theory of superconductivity, in particular the theory of Bardeen, Cooper, and Schrieffer (BCS), is subject of an upcoming second part of this textbook which is planned to be published directly after the present textbook.

After explaining the fundamental assumptions in great detail, all physical properties and effects in superconducting systems are derived from that. Great importance is placed on a detailed and comprehensible derivation of all formulas. A brief introduction to important concepts of classical physics (mechanics, electrodynamics, thermodynamics) and to the basic idea of quantum mechanics is given. All definitions and basic assumptions are explained in great detail, and all theoretical results are derived from these basic equations in such a way as the reader is able to follow the particular steps without additional literature. Final aim is to give the reader the ability to understand the basic experimental properties of superconductors.

An overview of these experimental findings is given in chapter 1 in form of a rather compact illustration. The theoretical explanations of these findings are then presented in a comprehensive way in the subsequent chapters 2-5. The following topics will be dealt with: Chapter 2 develops the basic properties of the London theory of superconductivity. Particular attention is given to the explanation of the famous Meissner effect and the theoretical description of the magnetic field inside a superconductor.

Chapter 3 gives an introduction to the basics of thermodynamics, which is needed for the concepts in this textbook. Starting with the laws of thermodynamics, the general concept of thermodynamic potentials is introduced and free energy and Gibbs free energy are defined. In this context, the concept of functional derivation is also discussed. The chapter ends with a basic introduction to the Landau theory of phase transitions.

Chapter 4 concentrates on the theory of the phase transition from the superconducting state to the normal state within the Ginzburg-Landau theory. Starting from the Landau theory of phase transitions, the concepts of order parameters and the minimization of the Landau potential are described. These considerations lead to a basic set of field equations describing the superconducting state as well as the magnetic field inside the superconductor. The equations are extensively derived and solved for specific cases. In this context, the characteristic length scales determining the superconducting state are also discussed.

Finally, in chapter 5, prominent applications of the Ginzburg-Landau theory are presented. Particular attention is given to the critical magnetic field. In this context, the chapter develops a systematic access to type 1 and type 2 superconductors and provides the derivation of the characteristic parameters to distinguish the two types of superconductors. Moreover, the lower and upper critical values of the magnetic field in type 2 superconductors are explicitly calculated. The chapter provides a comprehensive description of Abrikosov vortices and their relevance for the field penetration.

Throughout this book, we shall use Gaussian units. Relevant equations within this textbook are numbered and referred in the text by citing the corresponding equation number. All figures are numbered as well.

Chapter 1 – Characteristic properties

The defining property of a superconductor is the disappearance of electrical resistance at very low temperatures. Associated with the infinitely good conductivity is the ability to completely displace a magnetic field from the interior of the material. The superconductivity is found in all metals and also in other compounds. Therefore, it is a fundamental property of condensed matter that goes far beyond a presence only in some special compounds. The phenomenon was discovered by Onnes in 1911.

In this chapter an overview of the known experimental properties of superconductors is given. Starting with the basic phenomena of the vanishing electrical resistance in section 1.1 and Meissner effect in section 1.2, all further important effects are described in the sections 1.3-1.6.

1.1 – Electrical resistance

In an ordinary metal at room temperature, the electrical resistance is relatively small, but not zero. Usually, one measures the resistance by applying a bias voltage U and measuring the associated current I. The electrical resistance R is then defined by the ratio $R = U/I$, since the Ohm's law $I \propto U$ applies to metals at room temperature. If the same measurement is carried out at an extremely low temperature below a certain critical temperature, the current becomes 'infinitely large', so that the value zero is assigned to the resistance. Then it is said from an experimental point of view that the material is superconducting.

The critical temperature T_c below which the resistance vanishes (usually called transition temperature) depends strongly on material properties and external parameters. One example of a quantity which controls the superconducting transition very well is an externally applied magnetic field.

Figure 1.1 shows the electrical resistance of a superconductor (solid line) at low temperatures as typically measured by experiments. The central observation is a sharp transition from a typical behavior of the resistance of normal metals to unmeasurable small values if the temperature is cooled down below T_c. For comparison, the temperature behavior of a usual metal, where the superconducting transition has been suppressed (for example using magnetic fields), is also shown (dashed line).

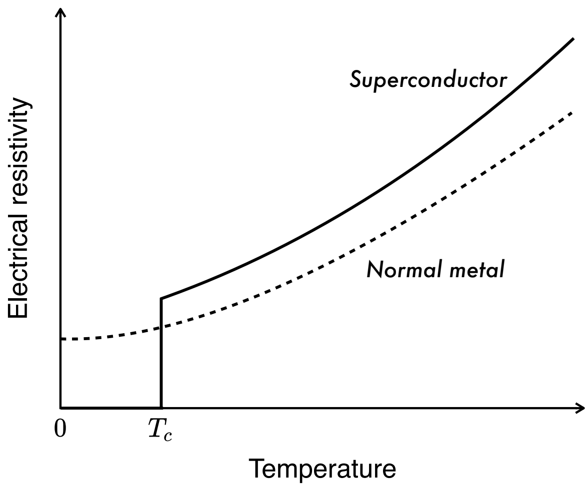

Figure 1.1: Schematic picture of the temperature behavior of a superconductor in comparison to a usual metal. Below a characteristic temperature the electrical resistance drops down to zero.

The property of the vanishing resistance was discovered during resistance measurements on mercury. The transition temperature for mercury is relatively small, $T_c = 4.2\,\mathrm{K}$, but the transition to a superconducting state has been found very soon also for other metallic compounds where the transition temperature might be slightly higher. Figure 1.2 shows measured values of transition temperatures for selected superconducting materials and the corresponding year of discovery. The conventional superconductors (circles) have T_c values up to $\approx 40\,\mathrm{K}$ for MgB_2. The well-known copper-based

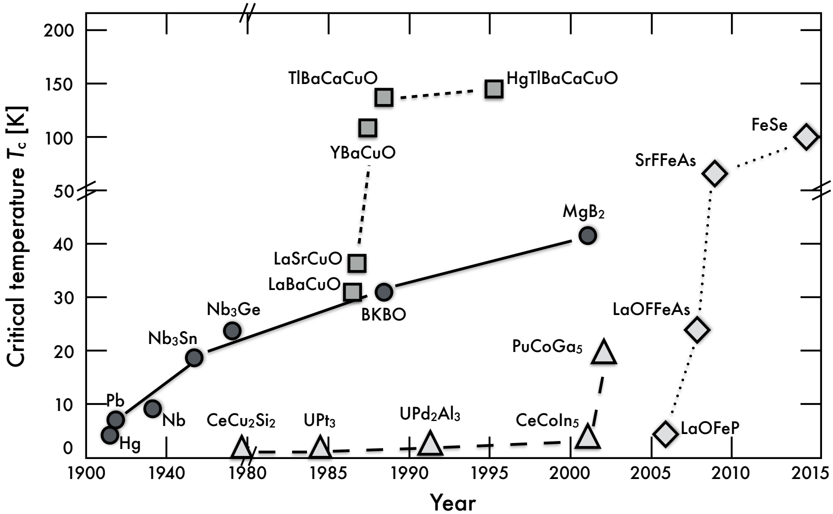

Figure 1.2: Timeline of the discovery of selected superconducting materials and the corresponding transition temperatures up to the year 2015. The conventional superconductors (circles) have transition temperatures up to 40 K. Higher values of T_c are possible for the copper-based superconductors (squares) and iron-based superconductors (diamonds). The heavy fermion superconductors (red triangles) are also counted to the unconventional superconductors despite their low T_c values.

high-temperature superconductors are characterized by relatively large critical temperatures up to 150 K at normal pressure.

Using a magnetic field which changes in time, it is possible to induce a steady current in a superconducting loop. This phenomenon is closely related to the vanishing electrical resistance. Experiments could not find any measurable reduction of the steady current over decades, i. e. the half-life is usually measured to be larger than 10^6 years.

1.2 – Meissner effect

If charges in the material can be displaced infinitely easily due to the lack of resistance, it is easy to imagine that they are also extremely sensitive to magnetic fields. The reason for this is that a small change in the magnetic field immediately leads to an induction of an electric field, to which the superconducting charges then react immediately with a large electric current. This current, in turn, generates a magnetic field that counteracts the external field. Experiments show that in most cases even the external field is completely displaced by this effect. This phenomenon, which occurs only in the superconducting state, is called the Meissner effect. It should be noted that metals in their normal state let the magnetic field almost completely into the material.

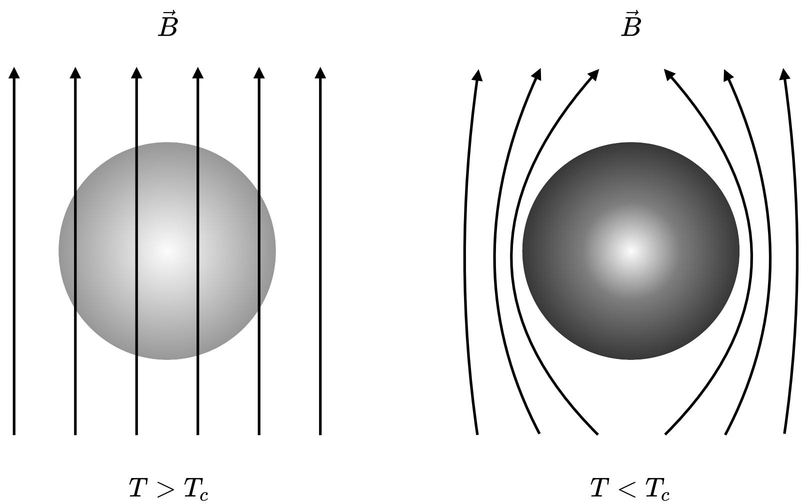

Figure 1.3: Meissner effect in a bulk superconductor. An external magnetic field $\overrightarrow{B}$ (field lines displayed by arrows) penetrates a metallic material (normal state, light gray). If the material is cooled down to temperatures below the transition temperature T_c (superconducting state, dark gray) the magnetic field is displaced so that the interior of the superconductor is field-free, $B = 0$.

Thus, as a consequence of the vanishing electrical resistance a superconductor strongly interacts with an external magnetic field. The effect is a displacement of the magnetic field from the interior of a superconductor during its transition to the superconducting state. A schematic picture of the Meissner effect is given in Figure 1.3. It was discovered in 1933 by Meissner and Ochsenfeld from measurements of the magnetic field distribution outside superconducting tin and lead samples.

The reaction of a material to an external magnetic field by the formation of its own magnetic field in the interior of the material is called *magnetization* of the material. The superconductor should thus have a very large magnetization directed against the external field. One speaks of ideal diamagnetism. As we will show in the following, the magnetization can be calculated without big effort if the Meissner effect is ideally realized, i. e. the magnetic field is completely displaced from the material. The complete displacement is of course an idealization, but we can very easily calculate further magnetic quantities, such as the magnetic susceptibility, which can be determined experimentally.

We consider a superconducting material that is placed in a homogeneous external magnetic field $\overrightarrow{H}$. Perfect realization of the Meissner effect means that the magnetic induction $\overrightarrow{B}$ is zero in the whole material volume. From the general relation $\overrightarrow{B} = \overrightarrow{H} + 4\pi\overrightarrow{M}$ between external field $\overrightarrow{H}$ and magnetization $\overrightarrow{M}$ of the material, we immediately find the relation

$$\overrightarrow{M} = -\frac{1}{4\pi}\overrightarrow{H}, \tag{1.1}$$

which determines the magnetic susceptibility in the superconducting state. This quantity describes the response of the material to an external magnetic field $\overrightarrow{H}$. Comparing (1.1) with the defining equation $\overrightarrow{M} = \chi\overrightarrow{H}$ of the magnetic susceptibility χ we find in the superconducting state the value $\chi = -1/(4\pi)$.

It turns out that in real materials the Meissner effect is not realized perfectly, i. e. there is a finite magnetic induction within a narrow region close to the surface of the superconducting material. The magnetic induction in the interior of the superconducting material is screened by surface currents flowing inside the region where the field penetrates. This area has a typical spatial extension of the order of the so-called London penetration depth λ_L (see chapter

2). Typical values of λ_L are around $500\,\mathring{A}$. In the surface area, the magnetic induction $\overrightarrow{B}$ is non-zero but decays exponentially and approaches $\overrightarrow{B} = 0$ in the interior of the material (bulk superconductor) where perfect diamagnetism is found as discussed above. The typical behavior of the magnetic induction $\overrightarrow{B}$ as a function of the distance to the surface is illustrated schematically in Figure 1.4.

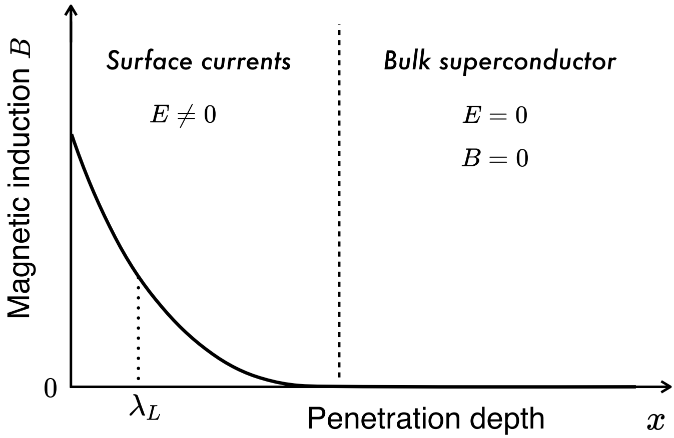

Figure 1.4: Magnetic field as a function of the distance to the surface ($x = 0$) of the superconducting material. The field decays within a typical length scale $\lambda_L \approx 500\ \mathring{A}$. In this surface area, currents lead to a screening of the magnetic field inside the material.

A superconductor is an ideal conductor and therefore any finite electric field $\overrightarrow{E}$ causes an infinitely large electric current. Thus, inside a superconducting material energy conservation can only be fulfilled if the interior of the superconductor is free of any electric field, i. e. $\overrightarrow{E} = 0$. This applies, of course, to a state of thermodynamic equilibrium. Thus, Maxwell's law of induction,

$$\operatorname{rot}\overrightarrow{E} = -\frac{1}{c}\frac{\partial \overrightarrow{B}}{\partial t}$$

(c: speed of light in vacuum) leads for $\overrightarrow{E} = 0$ to a time-independent (static) magnetic field inside a superconductor.

1.3 – Critical magnetic field

If the superconductor is in an external magnetic field, the superconducting state initially remains stable as long as the field strength is not too large. A further increase in the field strength leads to a phase transition to the normal state. The superconducting state can either disappear completely, or a mixed state of superconducting and normal conducting domains is first formed, which is then replaced by the normal state in a second transition.

In this section, we want to collect important experimental findings on the critical values of the external magnetic field. Here the question of importance is under which conditions regarding the magnetic field the superconducting state is thermodynamically stable.

1.3.1 – Field energy

At first, let us start with a few energetic considerations. As discussed in the previous section, when applying an external magnetic field $\overrightarrow{H}$, the magnetic induction $\overrightarrow{B}$ inside the material is suppressed. Thus, there is a strong difference between $\overrightarrow{H}$ and $\overrightarrow{B}$. This occurs because the superconducting state can extremely easily induce currents on the surface whose magnetic field counteracts the external one. However, the induction of these currents and the associated dislocation of the magnetic field costs energy. If the magnetic field strength is relatively small, this energy loss is small compared to the gain in energy caused by the formation of the superconducting state. This will be shown later. Therefore, if the value of the magnetic field is smaller than some critical value, the gain in energy due to the superconducting state can be larger than the energy loss through the displacement of the field. As a result, the superconducting state is stable while the Meissner effect is present.

The amount of field energy which is 'removed' from the interior of the superconducting material can be calculated approximately as follows. If we assume that the London penetration depth λ_L is small compared to the typical dimension of the superconducting material, $\lambda_L \ll V^{1/3}$, where V is the sample volume, this energy is equivalent to the total field energy of the magnetic field inside the volume of the superconductor. In this limit it corresponds exactly to the field energy inside the space volume V if the superconductor did not exist. The related amount of field energy can be calculated from electrodynamics, $E_H = VH^2/(8\pi)$. Roughly speaking, if this energy loss E_H becomes (for particular high field strength) larger than the energy gain associated with the manifestation of the superconducting state over the normal state, the superconducting state becomes unstable.

1.3.2 – Phase transition

The above considerations lead us to the insight that at a given temperature $T < T_c$ the superconducting phase is thermodynamically stable if the external magnetic field H is lower than a certain critical magnetic field $H_c(T)$. For temperatures larger than T_c, only the metallic phase is stable. An approximate formula for $H_c(T)$, which is valid for most of the conventional superconductors, can be derived from experiments. It reads

$$H_c(T) = H_0 \left[1 - \left(\frac{T}{T_c} \right)^2 \right], \tag{1.2}$$

where H_0 is the critical field at zero temperature. The critical magnetic field curve $H_c(T)$ represents the phase boundary between the superconducting phase and the metallic phase and is qualitatively shown in Figure 1.5. Note that there are unconventional superconductors in which the phase transition line described by (1.2) is further divided into two lines. These so-called type 2 superconductors will be treated later.

Generally, one distinguishes different orders of phase transitions. A simple way to specify the order of the phase transition is to investigate the latent heat which is exchanged during this transiti-

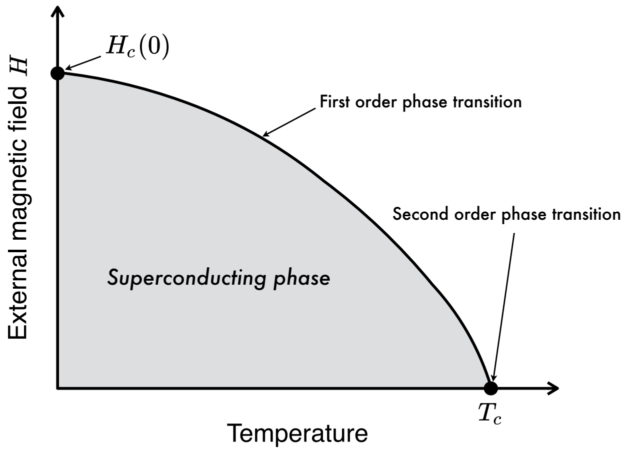

Figure 1.5: Schematic phase diagram of a conventional superconductor in the presence of an external magnetic field H. Except for $T = T_c$ and $T = 0$ the phase transition is of first order, i. e. a latent heat exchange is associated with the phase transition from the superconducting to the normal state.

on. If a finite amount of latent heat is measured, this indicates a first-order phase transition. In the case of a second-order phase transition, there would be no exchange of latent heat. A typical example of a first-order phase transition is the melting of water ice, where latent heat is supplied during the melting process.

In superconductors actual measurements reveal that for all transitions that take place at finite temperatures $T > 0$, which are less than the T_c of the field-free case, a latent heat is exchanged. The exchanged heat decreases more and more if the temperature, at which the transition takes place, is increased. Thus, along the black curve in the phase diagram in Figure 1.5 the phase transition is of first order. If the transition finally takes place exactly at T_c, i. e. without the presence of a magnetic field ($H = 0$), no latent heat is exchanged. This corresponds to a second-order phase transition. It is marked in Figure 1.5 with a point on the right edge of the phase boundary line. It can be shown theoretically that the phase transition at $T = 0$ on the left edge of the phase boundary line also corresponds to a second-order transition.

In summary, it can be said that in a finite magnetic field $H > 0$ and at finite temperature $T > 0$ the phase transition is of first order. In a field-free situation or at $T = 0$, it is of second order. We will describe and understand all the above-mentioned experimental results with the help of the theories to be developed.

1.3.3 – Type 1 and type 2 superconductors

As already mentioned above, we have to distinguish between two different types of superconductors with regard to the phase transition. This is closely linked to the typical length scales that occur in the variation of the particle density and the magnetic induction. In addition to the London penetration depth λ_L as the first important length scale, there is another one determining the properties of superconductors, the coherence length ξ. This quantity will be defined correctly in chapter 4. Here it should be only mentioned that ξ describes the characteristic length scale in which the superconducting wave function changes significantly. In addition to the London penetration depth, the length ξ can be used to further classify superconductors.

An important dimensionless parameter in this context is the *Ginzburg-Landau parameter*, defined by the ratio $\kappa := \lambda_L/\xi$. Depending on the value of κ, superconductors behave differently during the phase transition. This property is discussed in detail in chapter 5. In the following, a summary of the main results of this discussion is given.

In materials with $\kappa < 1/\sqrt{2}$, the transition takes place as described in the previous subsection: There is a critical magnetic field H_c at which the superconductivity breaks down. During the transition, the entire superconducting density disappears at the same time. The phase transition is of first order and accompanied by a latent heat transfer. Materials with $\kappa < 1/\sqrt{2}$ are called type 1 superconductors. In particular, materials with $\lambda_L \approx 0$ in which the Meissner effect is realized perfectly, belong to this family.

Materials with $\kappa > 1/\sqrt{2}$ are called type 2 superconductors. They are characterized by the existence of two critical field values, denoted by H_{c1} and H_{c2}. For external fields $H < H_{c1}$, the material behaves like a type 1 superconductor in the Meissner phase. For fields H larger than the so-called lower critical field H_{c1} the material enters an inhomogeneous superconducting phase. Here, the material has the property that a coexistence of superconducting and normal conducting regions in the material are observed. The material forms normal conducting and superconducting domains which coexist. The magnetic field partially penetrates into the material by flooding the normal conducting areas while it is still displaced from the superconducting areas.

In this phase, the magnetic flux passing through the normal conducting domains cannot be arbitrary, i. e. the field penetrates the material in a special way. It forms quantized tubes of magnetic flux which are formed by vortices of current (Abrikosov vortices). In this state, the current circulates around the normal conducting core of the vortex. Such structures were first described theoretically by Abrikosov in 1957 (Nobel Prize 2003).

The size of the Abrikosov vortex is described by the two length scales ξ and λ_L. While the core of the vortex has a characteristic size of the order of the coherence length ξ the currents flowing around the core decay on a distance of about λ_L. The circulating currents induce magnetic fields with the total flux equal to the flux quantum

$$\phi_0 = \frac{hc}{2e} = 2.07 \cdot 10^{-7}\,\mathrm{Gcm}^2, \tag{1.3}$$

where h is the Planck's constant, c the velocity of light, and e the elementary charge. Here,

$$\phi_0 = \int_{\mathcal{S}} \vec{B}\,\mathrm{d}\vec{A}$$

is the total magnetic flux of the circular current of one vortex, i. e. the surface $\mathcal{S}$ is chosen such that it encloses completely the current belonging to the particular vortex.

At H-values slightly above the lower critical field H_{c1}, the vortices arrange rather disordered. The vortex density increases with increasing magnetic field. An appropriate vortex model and the flux quantization is treated in chapter 5.

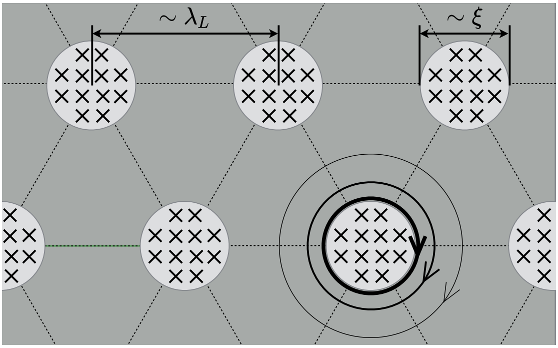

Figure 1.6: Schematic picture of the Abrikosov flux lattice in type 2 superconductors. An external magnetic field in the range $H_{c1} < H < H_{c2}$ penetrates the superconducting material (dark gray) in form of vortices (light gray). In the vortex the total magnetic flux is $\phi_0 = hc/(2e)$. The strength of the circular currents (indicated in the figure by the line thickness) decays for each vortex within a characteristic length of the order of the London penetration depth λ_L. This length scale determines also the distance between the vortex cores. For H-values close to H_{c2}, the vortices are arranged in form of a triangular lattice. The magnetic induction $\vec{B}$, schematically denoted by black crosses, points perpendicular to the plane of projection.

In a clean superconductor and for magnetic fields close to the upper critical field, the Abrikosov vortices arrange in form of a triangular lattice as illustrated in Figure 1.6. Each of the vortices carries one flux quantum. Note that, as with other lattices, defects may form as dislocations of the triangular Abrikosov lattice.

If the external magnetic field exceeds the upper critical field H_{c2}, the whole material volume enters the normal state. Typical values for the critical fields H_{c1} and H_{c2} are relatively large. For example, in $NbSn_3$ ($T_c = 18\,K$) experiments find

$H_{c1} \approx 200\,G$ and $H_{c2} \approx 100000\,G$.