

TABLE OF CONTENTS

	<u>Page</u>
<u>PREFACE</u>	I
<u>I. THE CANONICAL GEOMETRIC STRUCTURE OF RIGID SUPERSPACE AND SUSY TRANSFORMATIONS</u>	1
I.1 <u>Introduction to part I</u>	1
I.2 <u>Definition of rigid superspace</u>	2
(i) General definition	2
(ii) Rigid superspace as a reductive homogeneous space	3
(iii) The Riemannian view-point	5
I.3 <u>The left action on superspace</u>	5
(i) The method of induced representations ; definition of susy transformations	5
(ii) Action of the Lorentz group on rigid superspace	8
I.4 <u>Left-invariant vector fields on superspace</u>	9
(i) The right action	9
(ii) Invariance and covariance properties of (D_A)	10
(iii) Particularities of the anholonomic basis (D_A)	11
I.5 <u>Left-invariant forms on superspace</u>	12
(i) The dual basis (E^A)	12
(ii) Invariance properties of (E^A) : geometric characterization of susy transformations	14
(iii) Differential calculus with an anholonomic basis of 1-forms	16
I.6 <u>Canonical structures in the local formulation</u>	17
I.7 <u>Majorana spinor notation and transformation properties</u>	18
I.8 <u>Matrix representation of the SP-group and Maurer-Cartan forms</u>	20
(i) Matrix representation of the Poincaré group	20
(ii) Matrix representation of the SP-group	21
(iii) Maurer-Cartan forms on SP and their pullbacks to superspace	22
(iv) Left-invariant vector fields on SP and their projections to superspace	25

I.9	<u>Invariant connections on reductive homogeneous spaces</u>	26
(i)	The main result	27
(ii)	Local (superspace) expressions	30
(iii)	Is rigid superspace a flat space?	32
(iv)	Christoffel symbols of the canonical connection	33
I.10	<u>Rigid superspace from the Riemannian view-point</u>	34
(i)	Definition of a supermetric	34
(ii)	The Riemannian connection	36
(iii)	Relation between Killing vector fields and susy transformations	36
(iv)	Description of SYM-theories	38
I.11	<u>Miscellaneous facts</u>	39
(i)	Relation between the structure of superspace and constraints in susy theories	39
(ii)	The projection method applied to susy transformations and actions	41
(iii)	Rigid supersymmetry on non-trivial base manifolds	44
(iv)	Remarks on the generalizations to supergravity	44
II.	<u>THE GENERAL STRUCTURE OF SYM-THEORIES</u>	46
II.1	<u>The original approach and the WZ-supergauge</u>	46
II.2	<u>About formal gauge transformations</u>	50
II.3	<u>Geometric framework of SYM-theories</u>	51
(i)	Generalities	51
(ii)	The constraints on the superconnection	54
(iii)	Representation independent solution of the Bianchi identities	56
(iv)	Representation independent solution of the constraints	57
(v)	The different representations	58
II.4	<u>The gauge real representation</u>	59
(i)	Reality properties of the component and superfields	59
(ii)	Gauge and pregauge fixing ; the WZ gauge	62

II.5 The chiral/anti-chiral representations	63
(i) Superconnections and curvatures	64
(ii) Reality properties	65
(iii) Explicit solution of the constraints	67
(iv) Matter fields	68
(v) Projection to space-time	68
(vi) Starting from the (anti-) chiral representation	69
II.6 Summary	70
II.7 What's the use of all this?	78
III. CLASSICAL SYM-THEORIES IN THE GAUGE REAL REPRESENTATION	79
III.1 Component fields	79
III.2 The "susy transformations" and their algebra	80
(i) Definition of the transformations	80
(ii) Their algebra	81
III.3 The Lagrangian	82
(i) Massless matter and gauge fields	82
(ii) Mass terms	84
(iii) Analogy with the Aharonov-Bohm effect	84
IV. BRS-DIFFERENTIAL ALGEBRAS IN SYM-THEORIES	86
IV.1 General introduction to BRS differential algebras	86
(i) Historical note	86
(ii) From anticommuting variables to differential algebras	86
(iii) The "canonical formulation"	90
(iv) About anti-BRS transformations	92
(v) BRS differential algebras	93
(vi) Including space-time transformations	96
(vii) Associated Lie algebras and characterization of anomalies	98
(viii) Algebraic determination of anomalies	104

IV.2 BRS algebras and anomalies in SYM theories	108
(i) Superfield algebras in the chiral representation	108
(ii) BRS algebra and anomalies in the WZ-gauge	111
IV.3 Geometric approach to the "BRS algebra in the WZ-gauge"	114
(i) General framework	115
(ii) Superfield differential algebra	118
(iii) Projection to space-time	124
(iv) Summary	127
(v) Structure of the differential algebra and of the associated Lie algebra	128
(vi) Conclusion	129
V. GEOMETRY OF EXTENDED SUPERSYMMETRY	130
V.1 Geometry of rigid $U(N)$-superspace	130
V.2 General structure of extended SYM-theories	133
(i) Geometric framework	133
(ii) Constraints	134
(iii) Solution of the constraints and Bianchi identities	134
(iv) The gauge real representation ($N=2$)	135
V.3 $N=2$ BRS algebra in the WZ-gauge	137
APPENDICES	
A.1 Superspace conventions and notations (for $N=1$, $d=4$)	138
A.2 Complex (and hermitean) conjugation in simple supersymmetry	140
A.3 Complex conjugation in $N=2$ supersymmetry	143
A.4 Geometric interpretation of the canonical linear connection on reductive homogeneous spaces	144
A.5 Koszul's formula ("BRS cohomology")	146
A.6 On the description of anticommuting spinors in ordinary and supersymmetric field theories	150
References	167
Subject Index	177