

1	Introduction	1
2	Physical and Mathematical Background	5
2.1	States and Measurements	5
2.1.1	States	5
2.1.2	Measurements	6
2.2	Quantum Entanglement	8
2.2.1	Bipartite Entanglement	8
2.2.2	Multipartite Entanglement	16
2.3	Entanglement Witnesses	19
2.3.1	Bipartite Entanglement Witnesses	19
2.3.2	Schmidt Witnesses	24
2.3.3	Multipartite Entanglement Witnesses	25
2.4	Applications of Quantum Entanglement	27
2.4.1	Quantum Teleportation	27
2.4.2	Quantum Cryptography	29
2.4.3	Quantum Metrology	32
3	OSD Witnesses for Bipartite States	35
3.1	Theoretical Considerations	35
3.1.1	Construction for the Bipartite Case	35
3.1.2	Comparison with the Fidelity Witness	38
3.2	Optimization	39
3.2.1	Optimization with Respect to The OS Coefficients	39
3.2.2	Optimization with Respect to The Schmidt Operators	42
3.2.3	Practical Use of The Algorithms	49
3.3	Summary	56

4	The OSD Witness for the Multipartite Case	57
4.1	Construction for Multiparticle States	57
4.2	OSC Optimization in the Multipartite Case	59
4.2.1	Generalization of Algorithm 3.1	60
4.2.2	Further Approaches to Improve Fidelity Witnesses	65
4.2.3	Multiple Gradient Descent Algorithm	72
4.2.4	Short Summary	74
4.3	SO Optimization in the Multipartite Case	75
4.3.1	Generalization of Algorithm 3.2	75
4.3.2	Combination Strategies	78
4.3.3	Short Summary	81
5	Schmidt Number Witnesses	83
5.1	Construction of Schmidt Number Witnesses	83
5.1.1	Schmidt Number Witnesses $K = 3$	84
5.1.2	Schmidt Number Witnesses $K = 4$	86
5.1.3	Schmidt Number Witnesses $K > 4$	87
5.2	Example	91
5.3	Short Summary	92
6	Conclusion and Outlook	95
	Bibliography	99