

Contents

1	Basic Facts	1
1.1	Macroscopic Maxwell Equations	1
1.2	Magnetic Moment and Magnetization	7
1.3	Susceptibility	13
1.4	Classification of Magnetic Materials	15
1.4.1	Diamagnetism	15
1.4.2	Paramagnetism	15
1.4.3	Collective Magnetism	17
1.5	Elements of Thermodynamics	19
1.6	Problems	22
2	Atomic Magnetism	25
2.1	Hund's Rules	25
2.1.1	Russell–Saunders (<i>LS</i> -) Coupling	25
2.1.2	Hund's Rules for <i>LS</i> Coupling	27
2.2	Dirac Equation	28
2.3	Electron Spin	34
2.4	Spin–Orbit Coupling	40
2.5	Wigner–Eckart Theorem	45
2.5.1	Rotation	45
2.5.2	Rotation Operator	47
2.5.3	Angular Momentum	48
2.5.4	Rotation Matrices	50
2.5.5	Tensor Operators	52
2.5.6	Wigner–Eckart Theorem	53
2.5.7	Examples of Application	55
2.6	Electron in an External Magnetic Field	56
2.7	Nuclear Quadrupole Field	62
2.8	Hyperfine Field	67
2.9	Magnetic Hamiltonian of the Atomic Electron	72
2.10	Many-Electron Systems	74
2.10.1	Coulomb Interaction	74

2.10.2	Spin–Orbit Coupling	75
2.10.3	Further Couplings	76
2.11	Problems	81
3	Diamagnetism	85
3.1	Bohr–van Leeuwen Theorem	85
3.2	Larmor Diamagnetism (Insulators)	87
3.3	The Sommerfeld Model of a Metal	90
3.3.1	Properties of the Model	91
3.3.2	Sommerfeld Expansion	99
3.4	Landau Diamagnetism (Metals)	104
3.4.1	Free Electrons in Magnetic Field (Landau Levels)	104
3.4.2	Grand Canonical Potential of the Conduction Electrons	109
3.4.3	Susceptibility of the Conduction Electrons	117
3.5	The de Haas–Van Alphen Effect	121
3.5.1	Oscillations in the Magnetic Susceptibility	121
3.5.2	Electron Orbits in Magnetic Field	124
3.5.3	Physical Origin of the Oscillations	128
3.5.4	Onsager Argument	131
3.6	Problems	134
	References	136
4	Paramagnetism	137
4.1	Pauli Spin Paramagnetism	138
4.1.1	“Primitive” Theory of the Pauli Spin Paramagnetism	138
4.1.2	Temperature Corrections	141
4.1.3	Exchange Corrections	142
4.2	Paramagnetism of the Localized Electrons	155
4.2.1	Weak Spin–Orbit Interaction	158
4.2.2	Strong Spin–Orbit Coupling	164
4.2.3	Van Vleck Paramagnetism	166
4.3	Problems	171
	References	174
5	Exchange Interaction	175
5.1	Phenomenological Theories	178
5.1.1	The Exchange Field	178
5.1.2	Weiss Ferromagnet	180
5.2	Direct Exchange Interaction	184
5.2.1	Pauli’s Principle	184
5.2.2	The Heitler–London Method	188
5.2.3	Dirac’s Vector Model	195
5.3	Indirect Exchange Interaction	200
5.3.1	Rudermann–Kittel–Kasuya–Yosida (RKKY) Interaction	200
5.3.2	Superexchange	209

5.3.3	Double Exchange	217
5.4	Problems	226
	References	231
6	Ising Model	233
6.1	The Model	234
6.2	The One Dimensional Ising Model	236
6.2.1	Spontaneous Magnetization	236
6.2.2	One Dimensional Ising Model in External Field	240
6.3	The Phase Transition of Two-Dimensional Ising Model	245
6.3.1	The Method of Proof	245
6.3.2	Finite Ising Lattice with Special Boundary Conditions ...	247
6.3.3	Probabilities	247
6.3.4	Realization Possibilities for the Polygons	249
6.3.5	Magnetization of the Finite Lattice	252
6.3.6	Thermodynamic Limit	252
6.4	The Free Energy of the Two-Dimensional Ising Model	254
6.4.1	High-Temperature Expansion	255
6.4.2	Spin Products as Graphs	256
6.4.3	Loops	257
6.4.4	Directed Paths	261
6.4.5	Matrix M_1	264
6.4.6	Free Energy per Spin	266
6.4.7	Curie Temperature T_c	267
6.4.8	Specific Heat	268
6.4.9	Spontaneous Magnetization	269
6.5	Problems	270
	References	271
7	Heisenberg Model	273
7.1	Model Hamiltonian	273
7.1.1	Spin Operators	273
7.1.2	Model Extensions	279
7.2	Exact Statements	283
7.2.1	Mermin–Wagner Theorem	283
7.2.2	One-Magnon States of a Ferromagnet	291
7.3	Molecular Field Approximations	296
7.3.1	Ferromagnet	297
7.3.2	Antiferromagnet	303
7.3.3	Ferrimagnet	317
7.4	Spin Waves	322
7.4.1	Linear Spin Wave Theory for the Isotropic Ferromagnet .	322
7.4.2	“Renormalized” Spin Waves	329
7.4.3	Harmonic Approximation for Antiferromagnets	336
7.4.4	Harmonic Approximation for a Ferromagnet with Dipolar Interaction	345

7.5	Thermodynamics of $S = 1/2$ Ferromagnet.....	351
7.5.1	Tyablikov Decoupling	351
7.5.2	Spontaneous Magnetization	354
7.5.3	Thermodynamic Potentials	359
7.6	Thermodynamics of $S \geq 1/2$ Ferromagnets	361
7.6.1	Green's Functions	361
7.6.2	Spontaneous Magnetization	363
7.6.3	The Callen Method	371
7.7	Problems	381
	References	386
8	Hubbard Model	387
8.1	Introduction	387
8.2	Model for Band Magnets	388
8.2.1	Solid as a Many-Body System	388
8.2.2	Electrons in Narrow Energy Bands	389
8.2.3	Hubbard Model	393
8.3	Stoner Model	395
8.3.1	Stoner Ansatz (Ferromagnet)	395
8.3.2	Stoner Excitations	397
8.3.3	Magnetic Phase Transition	398
8.3.4	Static Susceptibility	405
8.4	Exact Statements and General Properties	409
8.4.1	Mermin–Wagner Theorem	410
8.4.2	The Infinitely Narrow Band	415
8.4.3	The Two-Site Model	420
8.4.4	The Exactly Half-Filled Band.....	427
8.4.5	Strong-Coupling Regime	431
8.4.6	Spectral Moments	437
8.4.7	High-Energy Expansions	439
8.4.8	Weak-Coupling Regime	441
8.4.9	Infinite Dimensions	444
8.4.10	Effective “impurity”-Problem.....	448
8.5	Magnetism and Electronic Correlations	451
8.5.1	Hubbard-I Approximation	451
8.5.2	Interpolation Method	454
8.5.3	Correlation Effects and Ferromagnetism	455
8.5.4	Criterion for Ferromagnetism	457
8.5.5	Static Susceptibility and Ferromagnetism	461
8.5.6	Spin-Dependent Band Shift	464
8.5.7	Quasiparticle Damping	470
8.5.8	Dynamical Mean Field Theory	475
8.5.9	Modified Perturbation Theory	479

8.5.10	Curie Temperature, Magnetization and Static Susceptibility	482
8.6	Problems	485
	References	490
A	Second Quantization	491
A.1	Identical Particles	492
A.2	Continuous Fock Representation	494
A.2.1	Symmetrized Many-Particle States	494
A.2.2	Construction Operators	495
A.2.3	Many-Body Operators	497
A.3	Discrete Fock Representation (Occupation Number Representation)	501
A.3.1	Symmetrized Many-Particle States	501
A.3.2	Construction Operators	503
A.4	Examples	506
A.4.1	Bloch Electrons	506
A.4.2	Wannier Electrons	508
A.4.3	Density Operator	509
A.4.4	Coulomb Interaction	510
A.5	Problems	511
B	The Method of Green's Functions	515
B.1	Linear Response Theory	515
B.1.1	Kubo Formula	515
B.1.2	Magnetic Susceptibility	518
B.1.3	Dielectric Function	520
B.2	Spectroscopies and Spectral Densities	523
B.3	Double-Time Green's Functions	528
B.3.1	Definitions and Equations of Motion	528
B.3.2	Spectral Representations	531
B.3.3	Spectral Theorem	534
B.3.4	Spectral Moments	536
B.3.5	Kramer's–Kronig Relations	537
B.3.6	Simple Applications	539
B.4	The Quasiparticle Concept	545
B.4.1	Interacting Electrons	545
B.4.2	Electronic Self-energy	548
B.4.3	Quasiparticles	551
B.4.4	Quasiparticle Density of States	555
B.4.5	Thermodynamics	558
B.5	Problems	559
C	Solutions to Problems	563
	Index	743