1

Mass Balances

1.1   Introduction

A mass balance is an accounting for mass flows and changes in inventory of mass for a system. When there is no generation (or usage) of material within the system: accumulation = input – output.

Lavoisier formulated the conservation law of mass for chemical reactions for the first time in 1775: the sum of the masses of the substances entering into a reaction equals the sum of the masses of the products of the reaction. He formulated his law after studying the formation of mercuric oxide and its decomposition. On oxidizing mercury to orange mercuric oxide, the mass increased because of the reaction with oxygen. On decomposing the compound, the original mass of mercury was obtained again.

Mass balance calculations are almost invariably a prerequisite to all other calculations in the solution of chemical engineering problems.

Balances can be made on many quantities different from mass, for example, on pounds sterling. Most of the principles are based on common sense and logical thinking.

In plants, on making a mass balance for a mixing point, a piece of equipment, a part of a plant, or a plant, it is important that the entering and the leaving flows are measured independently from each other. It is not a good practice to calculate a leaving flow from an entering flow or vice versa. If, still and all, this is done, it is advised to indicate this clearly.

In chemical plants, sometimes, there are parts which operate continuously while other parts function batchwise. For mass balances, both for design and for an analysis of a functioning plant, it is important to choose a unit of time and establish both the entering and leaving masses.

The law of conservation of mass is valid for process plants. The law is not valid for nuclear reactions.

Example 1.1

John is skating on the Dutch canals. After a while, he wants to take a rest. He enters a coffee-house and asks for a mug with hot milk. He is being served but then changes his mind. Could he give the milk back and get a mug with hot chocolate instead? It will give him calories for the remainder of the tour. No problem and he gets his hot chocolate. John enjoys the drink and soon after that gets on his feet again and is ready to leave. However, the proprietor addresses him: "Sir, could you pay please?"

JOHN: "Pay for what?"
PROPRIETOR: "For the hot chocolate!"
JOHN: "But I gave you the milk back for the chocolate."
PROPRIETOR: "Yes, but you did not pay the milk either!"
JOHN: "Of course I did not pay the milk because I did not drink it."

The proprietor is confused and John leaves the coffee-house. Who is right?
    Approach 1
    Make a mass balance.
    John enters with an empty stomach. He leaves with chocolate in his stomach. He must pay for the chocolate. The discussion is all eyewash.
    Approach 2
    Find the mistake in the reasoning.
    John uses the milk as a means of exchange. However, the milk does not belong to him. He must pay for the hot chocolate.

Example 1.2

This example comes from the United States.
    Three guys rent a hotel room and are charged $30 ($10 each). It occurred a while ago. They walk across the street and while there the hotel manager decides he charged them too much. The manager gives the bell boy $5 and tells him to return it to the three men. The bell boy, being practical and a bit dishonest, thinks: how can they split $5. The bell boy decides to keep $2 and give them each $1 back. Now the dilemma: the men each receives back $1 meaning they have now paid $9 each. Three times nine is $27 plus the $2 the bell boy has makes $29. What happened to the other $1? See Figure 1.1.

Considerations relevant for Example 1.2

Approach 1
Make a dollar balance for each of the three parties concerned.
Men: have spent 27.
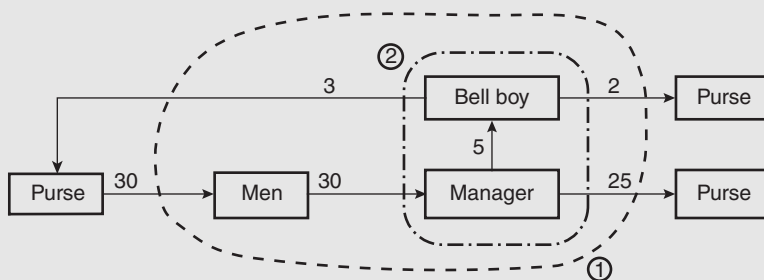Manager: received 25.
Bell boy: received 2.
27 = 25 + 2
There is no one-dollar issue. The discussion is idle.

Approach 2
Find the mistake in the reasoning.
Make a block scheme.

Circumscription ①: 30 = 3 + 2 + 25
Circumscription ②: 30 = 3 + 2 + 25

It is impossible to make a circumscription where 27 and 2
appear at the same side of the equation.

Figure 1.1   Sleeping in a hotel in the United States.

1.2 Theory

In a steady-state process, neither accumulation nor depletion occurs.

When a chemical reaction does not occur, an independent equation can be written to balance each compound. If a chemical reaction occurs, the component material balances have to be based on the elements themselves (H, O, S, etc.) or their equivalents (H_2, O_2, S_4, etc.).

A material balance problem has a unique solution only if the number of independent equations and the number of unknown variables are equal.

Example 1.3

A mixture of benzene, toluene, and xylene is fed to a distillation column. The feed rate is 10,000 kg · h^{-1}. The feed contains 60% by weight of benzene, 30% by weight of toluene, and 10% by weight of xylene. See Figure 1.2.

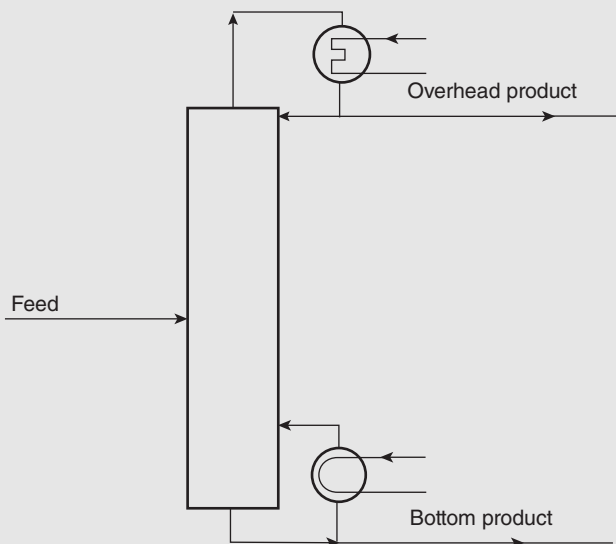**Figure 1.2** A distillation column.

Number of variables (3 flows and 3 components)	: 9
Number of independent mass balance equations	: 3
Number of fixed variables for a unique solution	: 9 − 3 = 6
Number of variables already specified	: 3
Number of variables to be specified by the designer	: 6 − 3 = 3

Specified by the designer: 5 kg · h^{-1} of toluene and 3 kg · h^{-1} of xylene in the overheads and 5 kg · h^{-1} of benzene in the bottoms.

Benzene in feed − benzene in bottoms = benzene in overheads

$0.6 \cdot 10{,}000 - 5 = 5{,}995$ kg · h^{-1}

Toluene in feed − toluene in overheads = toluene in bottoms

$0.3 \cdot 10{,}000 - 5 = 2{,}995$ kg · h^{-1}

Xylene in feed − xylene in overheads = xylene in bottoms

$0.1 \cdot 10{,}000 - 3 = 997$ kg · h^{-1}

Example 1.4

See Figure 1.3. The example concerns the air oxidation of cumene in a 0.75-l glass laboratory reactor. The oxidation proceeds at atmospheric pressure and 112 °C. When the air oxidation is carried out in the presence of an aqueous solution of sodium hydroxide, the reaction product is phenyl iso-propanol (cumyl alcohol).

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Material Balance — SUBJECT: Fenylisopropanol preparation

Scale: 0.75 l
Diameter reactor: 100 mm
Diameter stirrer: 60 mm   4 baffles d=10 mm
Stirrer type: InterMig 3x   Ne = 0.618 gassed cond.

Remarks :

Reduced NaOH intake, 0.14 kg NaOH-50/kg cumene

Subject nr. :   2427

IN	g	g cm^{-3}	°C	cm^3		OUT	g	g cm^{-3}	°C	cm^3
NaOH-50%	84	1.525		55.08						
Cumene (fresh)	283.5	0.865		327.7						
Cumene (recycle)	315	0.865		364.2						
CHP-K80	5.3	1.05		5.048						

Charging/heating

T = Amb. ⟶ 112 °C
t = 15 min.
688 g ~ 753 ml
970 rpm ⟶ 2.5 kW m^{-3}

Exhaust gas O$_2$ max 9.5%

Air max. 38 n-dm^3 h^{-1}

Water removal — 27.2
during first 270 minutes

Reaction

T = 112 °C
t = 480 min.
715 g ~ 925 ml(gassed)
970 rpm ⟶ 2.5 kW m^{-3}

Oxygen consumption during 480 min. is appr. 55 g

Figure 1.3 Laboratory batch air oxidation of a hydrocarbon.

Example 1.5

Coal can be converted into more convenient liquid products. H_2 and CO are the main gases that can be generated from in situ (in the ground) coal combustion in the presence of steam (as occurs naturally in the presence of ground water) [1]. The gaseous mixture is called synthesis gas because it can be used for the manufacture of various products, for example, methanol. The equation for the making of the latter material is: $CO + H_2 \rightarrow CH_3OH$. The conditions in a packed bed reactor are 50–100 bar and 240–260 °C. The reactor is filled with metal compounds that catalyze the reaction. Figure 1.4 contains a block scheme. The fresh feed is mixed with a recycle flow. Note that the fresh feed also contains some methane, CH_4. The feed to the reactor is the combination of these two flows. The reaction occurs and the gross product is separated into the net product and the recycle. The net product is methanol only. A purge is needed because otherwise methane would build up in the system. The methane level in the recycle and the purge is 3.2 mol%. The once-through conversion of CO in the reactor is 18%. The composition of the feed in mol% is in Figure 1.4. There is a small excess of H_2. The consequence of having a purge is that also some H_2 and CO are removed from the plant.

All flows and all compositions in the figure are calculated.

Solution

Take 100 kmol of fresh feed as a basis. Choose to make overall H_2-, C-, and O-balances. The unit of E, R, x, y, and z is kmol.

It immediately follows that $z = 0.2$ as there can be no buildup of CH_4. The mole balances are as follows.

$H_2 : 67.3 + 0.2 \cdot 2 = 2E + x + 2 \cdot 0.2.$

$C : 32.5 + 0.2 = E + y + 0.2.$

$O : 32.5 = E + y.$

This set of three equations cannot be solved as the latter two equations are identical. However, we have a further equation concerning the level of methane in the recycle and in the purge.

$$\frac{0.2}{x + y + 0.2} = 0.032.$$

It follows: $E = 31.25.$

$x = 4.8.$

$y = 1.25.$

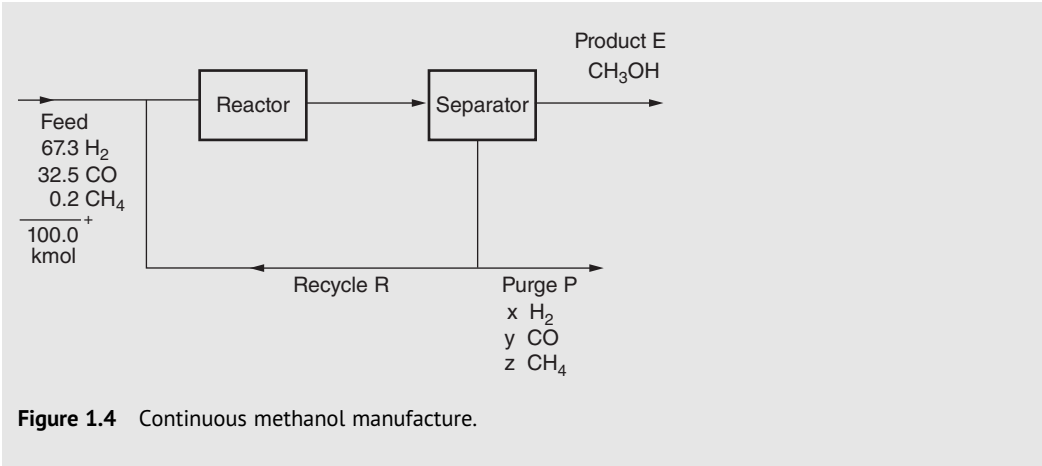The once-through balance is used to determine R. The mole fraction of CO in R is:

$$\frac{1.25}{4.8 + 1.25 + 0.2} = 0.2.$$

And an equation for the conversion in the reactor is:

$(32.5 + 0.2 \cdot R)(1 - 0.18) = 1.25 + 0.2 \cdot R,$

$R = 705,$

$P = 6.25.$

Figure 1.4 Continuous methanol manufacture.

1.3 Additional Material

In a plant, making a mass balance was used to establish what was wrong in the plant. It concerned a drying operation in a flash dryer. The name flash dryer was given to the dryer because the drying occurs in approximately 1 s. See Figure 1.5. Hot air and wet product travel concurrently through the dryer and drying is accomplished during the transport of the material. Air is heated indirectly in a heat exchanger. At the top of the dryer, product and spent air are separated in a cyclone. $1{,}000 \; \text{kg} \cdot \text{h}^{-1}$ of wet filter cake could be dried in this dryer. However, gradually the capacity fell to $850 \; \text{kg} \cdot \text{h}^{-1}$. To attain the final moisture content, it appeared necessary to raise the gas exit temperature from 65 to $80\,^\circ\text{C}$. Because of product quality reasons, it was not possible to raise the air inlet temperature. First, the moisture content of the filter cake was checked. It appeared that it had not changed. It was speculated that the cake properties had changed, however, that could not be confirmed. The temperature and the amount of fresh air had not changed either. It was decided to make mass balances in $\text{kg} \cdot \text{h}^{-1}$ for both dry air and water vapor.

Entering flows	Air flow in	Evaporation	Sum
Dry air	10,627.8	0.0	10,627.8
Water vapor	42.5 +	295.0 +	337.5 +
	10,670.3	295.0	10,965.3
Leaving flow			
Dry air	11,005.3		
Water vapor	737.5 +		
	11,742.5		

The dry air flow leaving the dryer is slightly larger than the dry air flow entering the dryer. This can be explained by air ingress through the product inlet. The water flow leaving the dryer is more than twice the water vapor flow entering the dryer. This can be explained by the leakage of steam into the hot air flow in the heat exchanger. The heater was repaired and the dryer performance improved accordingly.
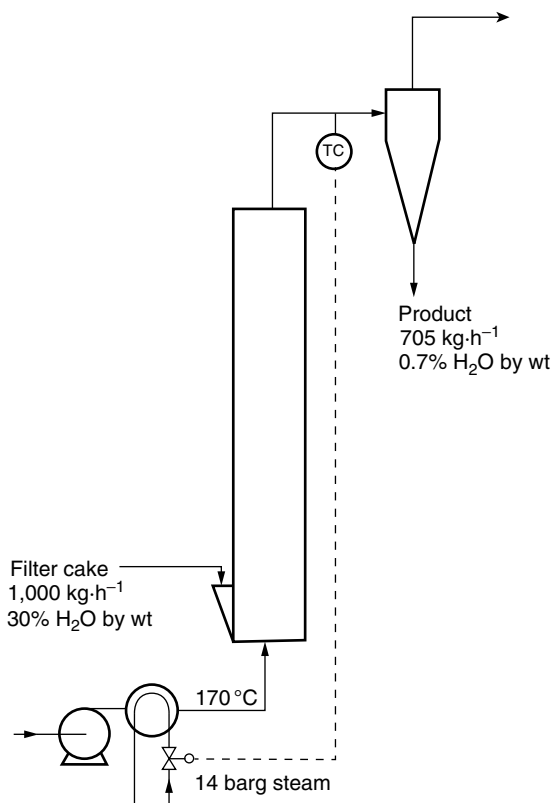
Figure 1.5 Flash drying.

Reference

1. Himmelblau, D. M., and Riggs, J. B. (2004). *Basic Principles and Calculations in Chemical Engineering* (pp. 369–371). Upper Saddle River, NJ: Prentice Hall.