

# Contents

<b>Chapter 1 Introduction—The Evolutive Finite Element Method</b>	1
1.1 Brief Review of the Features of Finite Element Method	1
1.2 Finite Element Method and Variational Principles	3
1.3 Research Areas of FEM	5
1.4 Advances in FEM and Outline of This Book	6
References	9

## PART I Advances in Variational Principles

<b>Chapter 2 The Sub-Region Variational Principles</b>	15
2.1 Introduction	15
2.2 The Sub-Region Variational Principle for Elasticity	16
2.3 The Sub-Region Variational Principle for Elastic Thin Plate	28
2.4 The Sub-Region Variational Principle for Elastic Thick Plate	40
2.5 The Sub-Region Variational Principle for Elastic Shallow Shell	51
2.6 The Sub-Region Mixed Energy Partial Derivative Theorem	58
References	64
<b>Chapter 3 Variational Principles with Several Adjustable Parameters</b>	66
3.1 Introduction	66
3.2 Several Patterns of Functional Transformation	67
3.3 Generalized Variational Principle Involving Several Adjustable Parameters	75
3.4 Variable-Substitution-Multiplier Method	83
References	85

## PART II Advances in Finite Element Method—Generalized Conforming Elements

<b>Chapter 4 Generalized Conforming Element Theory</b>	89
4.1 Introduction	89
4.2 Conforming and Nonconforming Elements—Some Consideration about “Conforming”	90
4.3 The First Pattern of Generalized Conforming Element—Replacing Nodal Conforming by Line Conforming Conditions	91

4.4	The Variational Basis of Generalized Conforming Element—Duality ...	94
4.5	The Synthesis of Energy Method and Weighted Residual Method —Flexibility .....	97
4.6	The Convergence of Generalized Conforming Element .....	99
	References .....	99

## **Chapter 5 Generalized Conforming Thin Plate Element I**

—	<b>Introduction</b> .....	101
5.1	Introduction .....	101
5.2	The Generalized Conforming Conditions and Their Equivalent Forms for Thin Plate Elements.....	102
5.3	General Formulations of the Generalized Conforming Thin Plate Elements .....	105
5.4	Several Construction Schemes of the Generalized Conforming Thin Plate Elements.....	107
5.5	A Collection of the Recent Generalized Conforming Thin Plate Elements .....	111
	References .....	118

## **Chapter 6 Generalized Conforming Thin Plate Element II**

—	<b>Line-Point and SemiLoof Conforming Schemes</b> .....	120
6.1	Line Conforming Scheme—Elements TGC-9 and TGC-9-1 .....	120
6.2	Line-Point Conforming Scheme—Rectangular Elements .....	130
6.3	Line-Point Conforming Scheme—Triangular Elements .....	146
6.4	Super-Basis Line-Point Conforming Scheme—Elements GCIII-R12 and GCIII-T9 .....	155
6.5	Super-Basis Point Conforming Scheme—Elements MB1-T9 and MB2-T9 .....	164
6.6	SemiLoof Conforming Scheme .....	167
	References .....	174

## **Chapter 7 Generalized Conforming Thin Plate Element III**

—	<b>Perimeter-Point and Least-Square Conforming Schemes</b> .....	176
7.1	Perimeter-Point Conforming Scheme—Elements LR12-1 and LR12-2 .....	176
7.2	The Application of Perimeter Conforming Conditions—Verification for the Convergence of the Element ACM .....	181
7.3	Super-Basis Perimeter-Point Conforming Scheme—Verification and Improvement of the Element BCIZ .....	187
7.4	Least-square Scheme—Elements LSGC-R12 and LSGC-T9 .....	198
	References .....	202

<b>Chapter 8 Generalized Conforming Thick Plate Element</b> .....	203
8.1 Summary of the Thick Plate Theory .....	203
8.2 Comparison of the Theories for Thick Plates and Thin Plates .....	215
8.3 Thick/Thin Beam Element .....	232
8.4 Review of Displacement-based Thick/Thin Plate Elements .....	235
8.5 Generalized Conforming Thick/Thin Plate Elements (1) — Starting with Assuming $(\psi, \gamma)$ .....	237
8.6 Generalized Conforming Thick/Thin Plate Elements (2) — Starting with Assuming $(w, \gamma)$ .....	249
8.7 Generalized Conforming Thin/Thick Plate Elements — From Thin to Thick Plate Elements .....	260
References .....	266
 <b>Chapter 9 Generalized Conforming Element for the Analysis of the Laminated Composite Plates</b> .....	268
9.1 Introduction .....	268
9.2 Fundamental Theory .....	270
9.3 New Element CTMQ20 for the Analysis of Laminated Composite Plates .....	275
9.4 The Hybrid-Enhanced Post-Processing Procedure for Element Stresses .....	286
9.5 Vibration Analysis of Laminated Composite Plates .....	290
9.6 Numerical Examples .....	292
References .....	301
 <b>Chapter 10 Generalized Conforming Element for the Analysis of Piezoelectric Laminated Composite Plates</b> .....	304
10.1 Introduction .....	304
10.2 The First-Order Shear Deformation Theory of Piezoelectric Laminated Composite Plate .....	306
10.3 New Piezoelectric Laminated Composite Plate Element CTMQE .....	309
10.4 The “Partial Hybrid”-Enhanced Post-Processing Procedure for Element Stresses .....	314
10.5 Numerical Examples .....	318
References .....	323
 <b>Chapter 11 Generalized Conforming Membrane and Shell Elements</b> .....	325
11.1 Introduction .....	325
11.2 Generalized Conforming Isoparametric Membrane Element .....	326
11.3 Membrane Elements with Drilling Freedoms—Definition of the Drilling Freedom and the Corresponding Rectangular and Quadrilateral Elements .....	334

11.4	Membrane Elements with Drilling Freedoms—Triangular Elements...	346
11.5	Flat-Shell Elements—Triangular Thick/Thin Shell Element GMST18.....	357
11.6	Shallow Shell Element—Variational Principle and Membrane Locking Problem.....	370
11.7	Shallow Shell Element—Triangular Element SST21 with Mid-Side Nodes.....	375
11.8	Shell Element for Geometrically Nonlinear Analysis —Triangular Flat-Shell Element GMST18 .....	382
11.9	Shell Element for Geometrically Nonlinear Analysis —Rectangular Shallow Shell Element SSR28 .....	386
	References .....	398

### **PART III Other Advances in Finite Element Method**

<b>Chapter 12</b>	<b>Sub-Region Mixed Element I—Fundamental Theory and Crack Problem .....</b>	<b>405</b>
12.1	Review of the Sub-Region Mixed Element Method .....	405
12.2	Basic Equations of the Sub-Region Mixed Element Method.....	408
12.3	2D Crack Problem.....	411
12.4	Cracked Thick Plate Problem.....	418
12.5	Surface Crack Problem in a 3D Body .....	426
	References .....	435

<b>Chapter 13</b>	<b>Sub-Region Mixed Element II—V-Notch Problem .....</b>	<b>438</b>
13.1	Introduction.....	438
13.2	Plane V-Notch Problem.....	438
13.3	Plane V-Notch Problem in a Bi-Material .....	450
13.4	Anti-Plane V-Notch Problem in a Bi-Material .....	457
13.5	V-Notch Problem in Reissner Plate.....	463
13.6	3D V-Notch Problem.....	481
	References .....	493

<b>Chapter 14</b>	<b>Analytical Trial Function Method I—Membrane and Plate Bending Elements.....</b>	<b>495</b>
14.1	Recognition of the Analytical Trial Function Method.....	495
14.2	4-Node Membrane Elements Based on the Analytical Trial Function Method .....	498
14.3	Avoiding Trapezoidal Locking Phenomenon by ATF Elements.....	500
14.4	The Basic Analytical Solutions of the Thick Plate Theory and ATF Elements Free of Shear Locking .....	504
14.5	Development of Quadrilateral Thin-Thick Plate Element Based on the Analytical Trial Function Method.....	506

14.6	Analytical Trial Function Method for Developing a Triangular Thick Plate Element Based on a Thin Plate Element .....	510
	References .....	516
<b>Chapter 15 Analytical Trial Function Method II—Singular Elements with Crack and Notch .....</b>		
15.1	Introduction .....	518
15.2	The Basic Analytical Solutions of the Plane Crack Problem .....	519
15.3	Element ATF-MS with Crack Formulated by the Analytical Trial Function Method.....	523
15.4	Error Analysis of Element ATF-MS with Crack.....	525
15.5	Analysis of Zero Energy Mode in Element and in Structural System .....	529
15.6	The Basic Analytical Solutions of the Plane Notch Problem .....	535
15.7	Element ATF-VN with Notch Formulated by the Analytical Trial Function Method.....	538
15.8	Error Analysis of Element ATF-VN with Notch .....	542
	References .....	545
<b>Chapter 16 Quadrilateral Area Coordinate Systems, Part I —Theory and Formulae .....</b>		
16.1	Introduction .....	546
16.2	The Isoparametric Coordinate Method and the Area Coordinate Method .....	547
16.3	Two Shape Characteristic Parameters of a Quadrilateral .....	549
16.4	The Definition of Quadrilateral Area Coordinates (QACM- I).....	553
16.5	Two Identical Relations Among Area Coordinates (QACM- I) .....	556
16.6	Transformation Relations Between the Area Coordinate System (QACM- I) and the Cartesian or Isoparametric Coordinate System ..	558
16.7	Differential Formulae (QACM- I) .....	560
16.8	Integral Formulae (QACM- I) .....	562
16.9	The Proof of the Basic Formulae (A) and (B) (QACM- I) .....	565
16.10	The Proof of the Basic Formulae (C) (QACM- I) .....	569
16.11	The Quadrilateral Area Coordinate System with Only Two Components (QACM-II) .....	570
	References .....	580
<b>Chapter 17 Quadrilateral Area Coordinate Systems, Part II —New Tools for Constructing Quadrilateral Elements .....</b>		
17.1	Introduction .....	582
17.2	Sensitivity Analysis of Isoparametric Elements to Mesh Distortion ...	583
17.3	Brief Review of the Finite Element Models Formulated by Quadrilateral Area Coordinate Methods.....	586

17.4	4-Node Quadrilateral Membrane Elements Formulated by the Area Coordinate Method .....	589
17.5	Geometrically Nonlinear Analysis Using Element AGQ6- I .....	601
17.6	Quadrilateral Membrane Elements with Drilling Degrees of Freedom Formulated by the Area Coordinate Method.....	606
17.7	8-Node Quadrilateral Membrane Elements Formulated by the Area Coordinate Method .....	613
17.8	Quadrilateral Thin Plate Element Formulated by the Area Coordinate Method.....	620
17.9	Quadrilateral Thick Plate Element Formulated by the Area Coordinate Method.....	628
17.10	Quadrilateral Laminated Composite Plate Element Formulated by the Area Coordinate Method .....	635
	References.....	637

## **Chapter 18 Spline Element I— Analysis of High-Rise Building**

<b>Structures</b> .....	641
18.1 Introduction .....	641
18.2 Spline Beam Elements .....	642
18.3 Spline Plane Membrane Elements.....	646
18.4 Analysis of Shear Wall Structures by Spline Elements.....	648
18.5 Analysis of Frame-Tube Structures by Spline Elements.....	655
References.....	661

## **Chapter 19 Spline Element II — Analysis of Plate/Shell Structures** .....

19.1 Spline Elements for Thin Plate Bending .....	663
19.2 Spline Elements for Thick/Thin Beam and Plate .....	665
19.3 Spline Elements for Shallow Shell.....	670
19.4 Spline Elements for Thick/Thin Shell.....	672
19.5 Spline Elements for Geometrically Nonlinear Analysis.....	681
References.....	689

## **Chapter 20 Concluding Remarks**.....

20.1 Seven New Achievements in the Finite Element Method .....	691
20.2 Five New Element Series with 108 New Element Models .....	693
20.3 New Solution Strategies for Five Challenging Problems.....	699
References.....	700

## **Appendix** .....

A The equivalent equation of the functional stationary condition (2-45)....	703
B The node conditions derived from the stationary condition (2-77) .....	704
C $l_{ij}$ and $\gamma_{ij}$ in Eq. (13-137).....	705
D $s_{ij}$ and $t_{ij}$ in Eq. (13-144).....	706