

Contents

1. Relativistic Wave Equation for Spin-0 Particles:	
The Klein–Gordon Equation and Its Applications	1
1.1 The Notation	2
1.2 The Klein–Gordon Equation	4
1.3 The Nonrelativistic Limit	7
1.4 Free Spin-0 Particles	8
1.5 Energy-Momentum Tensor of the Klein–Gordon Field	12
1.6 The Klein–Gordon Equation in Schrödinger Form	21
1.7 Charge Conjugation	26
1.8 Free Spin-0 Particles in the Feshbach–Villars Representation	31
1.9 The Interaction of a Spin-0 Particle with an Electromagnetic Field	41
1.10 Gauge Invariance of the Coupling	49
1.11 The Nonrelativistic Limit with Fields	50
1.12 Interpretation of One-Particle Operators in Relativistic Quantum Mechanics	68
1.13 Biographical Notes	97
2. A Wave Equation for Spin-$\frac{1}{2}$ Particles: The Dirac Equation	99
2.1 Free Motion of a Dirac Particle	107
2.2 Single-Particle Interpretation of the Plane (Free) Dirac Waves	111
2.3 Nonrelativistic Limit of the Dirac Equation	120
2.4 Biographical Notes	126
3. Lorentz Covariance of the Dirac Equation	127
3.1 Formulation of Covariance (Form Invariance)	130
3.2 Construction of the $\hat{S}$ Operator for Infinitesimal Lorentz Transformations	140
3.3 Finite Proper Lorentz Transformations	143
3.4 The $\hat{S}$ Operator for Proper Lorentz Transformations	144
3.5 The Four-Current Density	147
3.6 Biographical Notes	148
4. Spinors Under Spatial Reflection	149
5. Bilinear Covariants of the Dirac Spinors	151
5.1 Biographical Notes	156

6. Another Way of Constructing Solutions of the Free Dirac Equation:	
Construction by Lorentz Transformations	157
6.1 Plane Waves in Arbitrary Directions	161
6.2 The General Form of the Free Solutions and Their Properties	165
6.3 Polarized Electrons in Relativistic Theory	174
7. Projection Operators for Energy and Spin	177
7.1 Simultaneous Projections of Energy and Spin	181
8. Wave Packets of Plane Dirac Waves	183
9. Dirac Particles in External Fields: Examples and Problems	197
10. The Two-Centre Dirac Equation	261
11. The Foldy–Wouthuysen Representation for Free Particles	277
11.1 The Foldy–Wouthuysen Representation in the Presence of External Fields	285
12. The Hole Theory	291
12.1 Charge Conjugation	299
12.2 Charge Conjugation of Eigenstates with Arbitrary Spin and Momentum	309
12.3 Charge Conjugation of Bound States	310
12.4 Time Reversal and PCT Symmetry	312
12.5 Biographical Notes	323
13. Klein’s Paradox	325
14. The Weyl Equation – The Neutrino	333
15. Wave Equations for Particles with Arbitrary Spins	347
15.1 Particles with Finite Mass	347
15.2 Massless Particles	355
15.3 Spin-1 Fields for Particles with Finite Mass: Proca Equations	359
15.4 Kemmer Equation	361
15.5 The Maxwell Equations	364
15.6 Spin- $\frac{3}{2}$ Fields	383
15.7 Biographical Notes	388
16. Lorentz Invariance and Relativistic Symmetry Principles	389
16.1 Orthogonal Transformations in Four Dimensions	389
16.2 Infinitesimal Transformations and the Proper Subgroup of $O(4)$	390
16.3 Classification of the Subgroups of $O(4)$	396
16.4 The Inhomogeneous Lorentz Group	398
16.5 The Conformal Group	400
16.6 Representations of the Four-Dimensional Orthogonal Group and Its Subgroups	402
16.6.1 Tensor Representation of the Proper Groups	402
16.6.2 Spinor Representations	403

16.7 Representation of $SL(2, C)$	406
16.8 Representations of $SO(3, R)$	407
16.9 Representations of the Lorentz Group L_p	408
16.10 Spin and the Rotation Group	410
16.11 Biographical Notes	415
Subject Index	417

Contents of Examples and Exercises

1.1	The Charged Klein–Gordon Field	11
1.2	Derivation of the Field Equations for Wavefields	13
1.3	Determination of the Energy-Momentum Tensor for a General Lagrange Density $\mathcal{L}(\psi_\sigma, \partial\psi_\sigma/\partial x^\mu)$	16
1.4	Lagrange Density and Energy-Momentum Tensor of the Schrödinger Equation	18
1.5	Lorentz Invariance of the Klein–Gordon Equation	20
1.6	C Parity	28
1.7	Lagrange Density and Energy-Momentum Tensor of the Free Klein–Gordon Equation in the Feshbach–Villars Representation .	34
1.8	The Hamiltonian in the Feshbach–Villars Representation	37
1.9	Solution of the Free Klein–Gordon Equation in the Feshbach–Villars Representation	39
1.10	Separation of Angular and Radial Parts of the Wave Function for the Stationary Klein–Gordon Equation with a Coulomb Potential .	44
1.11	Pionic Atom with Point-Like Nucleus	45
1.12	Lagrange Density and Energy-Momentum Tensor for a Klein–Gordon Particle in an Electromagnetic Field	51
1.13	Solution of the Klein–Gordon Equation for the Potential of an Homogeneously Charged Sphere	53
1.14	The Solution of the Klein–Gordon Equation for a Square-Well Potential	56
1.15	Solution of the Klein–Gordon Equation for an Exponential Potential	59
1.16	Solution of the Klein–Gordon Equation for a Scalar $1/r$ Potential ..	61
1.17	Basics of Pionic Atoms	65
1.18	Calculation of the Position Operator in the Φ Representation	73
1.19	Calculation of the Current Density in the Φ Representation for Particles and Antiparticles	76
1.20	Calculation of the Position Eigenfunctions in the Coordinate Representation	78
1.21	Mathematical Supplement: Modified Bessel Functions of the Second Type, $K_\nu(z)$	81
1.22	The Kisslinger Potential	83
1.23	Evaluation of the Kisslinger Potential in Coordinate Space	91
1.24	Lorentz–Lorenz Effect in Electrodynamics and Its Analogy in Pion–Nucleon Scattering (the Ericson–Ericson Correction)	92

2.1	Representation of the Maxwell Equations in the Form of the Dirac Equation	105
2.2	Lagrange Density and Energy-Momentum Tensor of the Free Dirac Equation	114
2.3	Calculation of the Energies of the Solutions to the Free Dirac Equation in the Canonical Formalism	116
2.4	Time Derivative of the Position and Momentum Operators for Dirac Particles in an Electromagnetic Field	121
3.1	Pauli's Fundamental Theorem: The 16 Complete 4×4 Matrices $\hat{F}_A$	132
3.2	Proof of the Coefficients $\hat{\sigma}_{\alpha\beta}$ for the Infinitesimal Lorentz Transformation	141
3.3	Calculation of the Inverse Spinor Transformation Operators	146
5.1	Transformation Properties of Some Bilinear Expressions of Dirac Spinors	154
5.2	Majorana Representation of the Dirac Equation	155
6.1	Calculation of the Spinor Transformation Operator in Matrix Form ..	162
6.2	Calculation of the Spinor $u(p + q)$ by Means of a Lorentz Transformation from the Spinor $u(p)$ of a Free Particle	164
6.3	Normalization of the Spinor $\omega^r(\mathbf{p})$	166
6.4	Proof of the Relation $\omega^{r\dagger}(\varepsilon_r \mathbf{p}) \omega^{r'}(\varepsilon_{r'} \mathbf{p}) = \delta_{rr'} (E/m_0 c^2)$	168
6.5	Independence of the Closure Relation from the Representation of the Dirac Spinors	171
6.6	Proof of Another Closure Relation for Dirac Spinors	172
8.1	The Gordon Decomposition	184
8.2	Calculation of the Expectation Value of a Velocity Operator	186
8.3	Calculation of the Norm of a Wave Packet	188
8.4	Calculation of the Current for a Wave Packet	190
8.5	Temporal Development of a Wave Packet with Gaussian Density Distribution	191
9.1	Eigenvalue Spectrum of a Dirac Particle in a One-Dimensional Square-Well Potential	197
9.2	Eigenvalues of the Dirac Equation in a One-Dimensional Square Potential Well with Scalar Coupling ..	206
9.3	Separation of the Variables for the Dirac Equation with Central Potential (Minimally Coupled)	210
9.4	Commutation of the Total Angular Momentum Operator with the Hamiltonian in a Spherically Symmetric Potential	215
9.5	A Dirac Particle in a Spherical Potential Box	217
9.6	Solution of the Radial Equations for a Dirac Particle in a Coulomb Potential	225
9.7	Discuss the Sommerfeld Fine-Structure Formula and the Classification of the Electron Levels in the Dirac Theory ...	231
9.8	Solution of the Dirac Equation for a Coulomb and a Scalar Potential	234
9.9	Stationary Continuum States of a Dirac Particle in a Coulomb Field ..	239
9.10	Muonic Atoms	249

9.11	Dirac Equation for the Interaction Between a Nuclear and an External Field Taking Account of the Anomalous Magnetic Moment	254
9.12	The Impossibility of Additional Solutions for the Dirac–Coulomb Problem Beyond $Z = 118$	257
12.1	Radiative Transition Probability from the Hydrogen Ground State to the States of Negative Energy	294
12.2	Expectation Values of Some Operators in Charge-Conjugate States ..	302
12.3	Proof of $\langle \hat{H}(-e) \rangle_c = -\langle \hat{H}(e) \rangle$	305
12.4	Effect of Charge Conjugation on an Electron with Negative Energy ..	306
12.5	Representation of Operators for Charge Conjugation and Time Reversal	307
12.6	Behaviour of the Current with Time Reversal and Charge Conjugation	318
13.1	Klein’s Paradox and the Hole Theory	329
14.1	Dirac Equation for Neutrinos	341
14.2	CP as a Symmetry for the Dirac Neutrino	342
14.3	Solutions of the Weyl Equation with Good Angular Momentum	344
15.1	Eigenvalue Equation for Multispinors	348
15.2	Multispinor $\omega^{(+)}$ as Eigenvector of $\hat{\Sigma}^2$	350
15.3	Multispinor of Negative Energy	351
15.4	Construction of the Spinor $\omega_{\alpha\beta\ldots\tau}^{(+)}(\mathbf{p}, i)$	352
15.5	The Bargmann–Wigner Equations	354
15.6	γ Matrices in the Weyl Representation	356
15.7	Commutation Relation of Kemmer Matrices	365
15.8	Properties of the Kemmer Equation Under Lorentz Transformation ..	368
15.9	Verification of the Kemmer Algebra	371
15.10	Verification of the Proca Equations from the Lagrange Density	372
15.11	Conserved Current of Vector Fields	373
15.12	Lorentz Covariance of Vector Field Theory	373
15.13	Maxwell-Similar Form of the Vector Fields	374
15.14	Plane Waves for the Proca Equation	375
15.15	Transformation from the Kemmer to the Proca Representation	377
15.16	Lagrange Density for Kemmer Theory	378
15.17	The Weinberg–Shay–Good Equations	378
15.18	Lagrangian Density for the Weinberg–Shay–Good Theory	381
15.19	Coupling of Charged Vector Mesons to the Electromagnetic Field ..	382
15.20	A Useful Relation	383
16.1	Transformation Relations of the Rest Mass Under Dilatations	401
16.2	D^1 Representation of SU(2)	405
16.3	Vector Representation and Spin	413