

Contents

Preface	xi
1 Definitions of Continuum Mechanics	1
1.1 Vectors and Tensors	1
1.1.1 Covariant Differentiation	5
1.1.2 The Levi-Civita Tensor	7
1.1.3 Differential Operations	9
1.1.4 Physical Components of Vectors and Tensors . . .	9
1.1.5 Eigenvalues and Eigenvectors of a Symmetric Tensor	10
1.1.6 The Ostrogradsky-Gauss Theorem	12
1.1.7 The Stokes Theorem	14
1.1.8 The Weyl Formula	15
1.2 Eulerian and Lagrangian Description of a Continuum: Strain Tensor	24
1.2.1 Lagrangian and Eulerian Description of a Continuum	24
1.2.2 Strain Tensor	28
1.2.3 A Condition for Compatibility of Deformations . .	35
1.2.4 Rate-of-Strain Tensor: Cauchy-Helmholtz Theorem	37
1.3 Stress Tensor	55
1.3.1 The Cauchy Stress Tensor in the Accompanying Coordinate System	55
1.3.2 Piola-Kirchhoff Stress Tensors in the Reference Frame and in the Eulerian Coordinates	59
1.3.3 Principal Values and Invariants of the Stress Tensor	61
1.3.4 Differentiation of the Stress Tensor with Respect to Time	63
References	73

2	Fundamental Principles and Laws of Continuum Mechanics	75
2.1	Equations of Continuity, Motion, and Energy for a Continuum	75
2.1.1	Continuity Equation	76
2.1.2	Equations of Motion and of Momentum Moment	78
2.1.3	The Energy Conservation Law: The First and Second Laws of Thermodynamics	84
2.1.4	Equation of State (General Relations)	92
2.1.5	Equations of an Ideal and Viscous, Heat-Conducting Gas	95
2.2	The Hamilton–Ostrogradsky’s Variational Principle in Continuum Mechanics	115
2.2.1	Euler–Lagrange Equations in Lagrangian Coordinates	115
2.2.2	Hamilton’s Equations in Lagrangian Coordinates	121
2.2.3	Euler–Lagrange Equations in Eulerian Coordinates and Murnaghan’s Formula	125
2.3	Conservation Laws for Energy and Momentum in Continuum Mechanics	135
2.3.1	Conservation Laws in Cartesian Coordinates	135
2.3.2	Conservation Laws in an Arbitrary Coordinate System	144
	References	152
3	The Features of the Solutions of Continuum Mechanics Problems	155
3.1	Similarity and Dimension Theory in Continuum Mechanics	155
3.2	The Characteristics of Partial Differential Equations	163
3.3	Discontinuity Surfaces in Continuum Mechanics	171
	References	185
4	Ideal Fluid	187
4.1	Integrals of Motion Equations of Ideal Fluid and Gas	187
4.1.1	Motion Equations in the Gromeka–Lamb Form	188
4.1.2	The Bernoulli Integral	188
4.1.3	The Lagrange Integral	189
4.2	Planar Irrotational Steady Motions of an Ideal Incompressible Fluid	193
4.2.1	The Governing Equations of Planar Flows	193
4.2.2	The Potential Flow past the Cylinder	202

4.2.3	The Method of Conformal Mappings	208
4.2.4	The Problem of the Flow around a Slender Profile	219
4.3	Axisymmetric and Three-Dimensional Potential Ideal Incompressible Fluid Flows	223
4.3.1	Axially Symmetric Flows	223
4.3.2	The Method of Sources and Sinks	231
4.3.3	The Program prog4-5.nb	233
4.3.4	The Transverse Flow around the Body of Revolution: The Program prog4-6.nb	235
4.4	Nonstationary Motion of a Solid in the Fluid	242
4.4.1	Formulation of a Problem on Nonstationary Body Motion in Ideal Fluid	242
4.4.2	The Hydrodynamic Reactions at the Body Motion	244
4.4.3	Equations of Solid Motion in a Fluid under the Action of Given Forces	247
4.5	Vortical Motions of Ideal Fluid	250
4.5.1	The Theorems of Thomson, Lagrange, and Helmholtz	250
4.5.2	Motion Equations in Friedmann's Form	257
4.5.3	The Biot-Savart Formulas and the Straight Vortex Filament	258
	References	265
5	Viscous Fluid	267
5.1	General Equations of Viscous Incompressible Fluid	268
5.1.1	The Navier-Stokes Equations	268
5.1.2	Formulation of Problems for the System of the Navier-Stokes Equations	275
5.2	Viscous Fluid Flows at Small Reynolds Numbers	276
5.2.1	Exact Solutions of the System of Equations for a Viscous Fluid	277
5.2.2	Viscous Fluid Motion between Two Rotating Coaxial Cylinders	280
5.2.3	The Viscous Incompressible Fluid Flow around a Sphere at Small Reynolds Numbers	282
5.3	Viscous Fluid Flows at Large Reynolds Numbers	287
5.3.1	Prandtl's Theory of Boundary Layers	288
5.3.2	Boundary Layer of a Flat Plate	293
5.4	Turbulent Fluid Flows	298
5.4.1	Basic Properties of Turbulent Flows	298
5.4.2	Laminar Flow Stability and Transition to Turbulence	300

5.4.3	Turbulent Fluid Flow	302
	References	309
6	Gas Dynamics	311
6.1	One-Dimensional Stationary Gas Flows	311
6.1.1	Governing Equations for Quasi-One-Dimensional Gas Flow	311
6.1.2	Gas Motion in a Variable Section Duct: Elementary Theory of the Laval Nozzle	313
6.1.3	Planar Shock Wave in Ideal Gas	321
6.1.4	Shock Wave Structure in Gas	329
6.2	Nonstationary One-Dimensional Flows of Ideal Gas	334
6.2.1	Planar Isentropic Waves	334
6.2.2	Gradient Catastrophe and Shock Wave Formation	342
6.3	Planar Irrotational Ideal Gas Motion (Linear Approximation)	346
6.3.1	Governing Equations and Their Linearization . . .	346
6.3.2	The Problem of the Flow around a Slender Profile	348
6.4	Planar Irrotational Stationary Ideal Gas Flow (General Case)	354
6.4.1	Characteristics of Stationary Irrotational Flows of Ideal Gas, Simple Wave: The Prandtl-Meyer Flow	355
6.4.2	Chaplygin's Equations and Method	366
6.4.3	Oblique Shock Waves	377
6.4.4	Interference of Stationary Shock Waves	382
6.5	The Fundamentals of the Gasdynamic Design Technology	386
6.5.1	The Basic Algorithm	387
6.5.2	The Superposition Procedure	391
6.5.3	The Complement Procedure	395
	References	399
7	Multiphase Media	401
7.1	Mathematical Models of Multiphase Media	403
7.1.1	General Equations of the Mechanics of Multiphase Media	403
7.1.2	Equations of a Two-Phase Medium of the Type of Gas-Solid Particles	407
7.1.3	Equations of a Bicomponent Medium of Gas Mixture Type	415

7.2 Correctness of the Cauchy Problem: Relations at Discontinuities in Multiphase Media 417

7.2.1 The Characteristics of a System of Equations for Gas-Particle Mixtures and Correctness of the Cauchy Problem 417

7.2.2 Jump Relations 431

7.3 Quasi-One-Dimensional Flows of a Gas-Particle Mixture in Laval Nozzles 442

7.3.1 The Equations of the Quasi-One-Dimensional Flow of a Gas-Particle Mixture 442

7.3.2 The Flow of a Gas-Particle Mixture in the Laval Nozzle with Small Velocity and Temperature Lags of Particles 447

7.4 The Continual-Discrete Model and Caustics in the Pseudogas of Particles 456

7.4.1 The Equations of the Continual-Discrete Model of a Gas-Particle Mixture at a Small Volume Concentration of Particles 456

7.4.2 Investigation of Caustics in the Pseudogas of Particles 460

7.5 Nonstationary Processes in Gas-Particle Mixtures 471

7.5.1 Interaction of a Shock Wave with a Cloud of Particles 471

7.5.2 Acoustic Approximation in the Problem of Shock Wave Interaction with a Particle's Cloud at a Small Volume Concentration 478

7.6 The Flows of Heterogeneous Media without Regard for Inertial Effects 486

7.6.1 The Brownian Motion of Particles in a Fluid 486

7.6.2 Fluid Filtration in a Porous Medium 493

7.7 Wave Processes in Bubbly Liquids 500

7.7.1 Equations of the Motion of a Bubbly Liquid 500

7.7.2 Equations for Weak Nonlinear Disturbances in Bubbly Liquids 507

7.7.3 Progressive, Weak Nonlinear Waves in Bubbly Liquids 512

References 522

Appendix A: *Mathematica* Functions 526

Appendix B: Glossary of Programs 550

Index 565