

Contents

1	Graph Theory and Small-World Networks	1
1.1	Random Graphs	1
1.1.1	The Small-World Effect	1
1.1.2	Basic Graph-Theoretical Concepts	3
1.1.3	Properties of Random Graphs	7
1.2	Generalized Random Graphs	11
1.2.1	Graphs with Arbitrary Degree Distributions	11
1.2.2	Probability Generating Function Formalism	15
1.2.3	Distribution of Component Sizes	17
1.3	Robustness of Random Networks	20
1.4	Small-World Models	23
1.5	Scale-Free Graphs	26
	Exercises	31
	Further Reading	31
2	Chaos, Bifurcations and Diffusion	33
2.1	Basic Concepts of Dynamical Systems Theory	33
2.2	The Logistic Map and Deterministic Chaos	38
2.3	Dissipation and Adaption	43
2.3.1	Dissipative Systems and Strange Attractors	43
2.3.2	Adaptive Systems	47
2.4	Diffusion and Transport	51
2.4.1	Random Walks, Diffusion and Lévy Flights	52
2.4.2	The Langevin Equation and Diffusion	55
2.5	Noise-Controlled Dynamics	57
2.5.1	Stochastic Escape	58
2.5.2	Stochastic Resonance	60
	Exercises	63
	Further Reading	64

3	Random Boolean Networks	67
3.1	Introduction	67
3.2	Random Variables and Networks	69
3.2.1	Boolean Variables and Graph Topologies	69
3.2.2	Coupling Functions	71
3.2.3	Dynamics	72
3.3	The Dynamics of Boolean Networks	74
3.3.1	The Flow of Information Through the Network	74
3.3.2	The Mean-Field Phase Diagram	76
3.3.3	The Bifurcation Phase Diagram	78
3.3.4	Scale-Free Boolean Networks	82
3.4	Cycles and Attractors	84
3.4.1	Quenched Boolean Dynamics	84
3.4.2	The $K = 1$ Kauffman Network	86
3.4.3	The $K = 2$ Kauffman Network	87
3.4.4	The $K = N$ Kauffman Network	88
3.5	Applications	91
3.5.1	Living at the Edge of Chaos	91
3.5.2	The Yeast Cell Cycle	92
3.5.3	Application to Neural Networks	94
	Exercises	96
	Further Reading	97
4	Cellular Automata and Self-Organized Criticality	99
4.1	The Landau Theory of Phase Transitions	99
4.2	Criticality in Dynamical Systems	104
4.2.1	$1/f$ Noise	107
4.3	Cellular Automata	108
4.3.1	Conway's Game of Life	109
4.3.2	The Forest Fire Model	110
4.4	The Sandpile Model and Self-Organized Criticality	112
4.5	Random Branching Theory	115
4.6	Application to Long-Term Evolution	119
	Exercises	126
	Further Reading	127
5	Statistical Modeling of Darwinian Evolution	129
5.1	Introduction	129
5.2	Mutations and Fitness in a Static Environment	131
5.3	Deterministic Evolution	134
5.3.1	Evolution Equations	135
5.3.2	Beanbag Genetics – Evolutions Without Epistasis	138
5.3.3	Epistatic Interactions and the Error Catastrophe	140
5.4	Finite Populations and Stochastic Escape	144
5.4.1	Strong Selective Pressure and Adaptive Climbing	145

5.4.2	Adaptive Climbing Versus Stochastic Escape	148
5.5	Prebiotic Evolution	149
5.5.1	Quasispecies Theory	149
5.5.2	Hypercycles and Autocatalytic Networks	151
5.6	Coevolution and Game Theory	154
	Exercises	158
	Further Reading	159
6	Synchronization Phenomena	161
6.1	Frequency Locking	161
6.2	Synchronization of Coupled Oscillators	162
6.3	Synchronization of Relaxation Oscillators	168
6.4	Synchronization and Object Recognition in Neural Networks	172
6.5	Synchronization Phenomena in Epidemics	175
	Exercises	178
	Further Reading	179
7	Elements of Cognitive Systems Theory	181
7.1	Introduction	181
7.2	Foundations of Cognitive Systems Theory	183
7.2.1	Basic Requirements for the Dynamics	183
7.2.2	Cognitive Information Processing Versus Diffusive Control	187
7.2.3	Basic Layout Principles	189
7.2.4	Learning and Memory Representations	191
7.3	Motivation, Benchmarks and Target-Oriented Self-Organization	195
7.3.1	Cognitive Tasks	195
7.3.2	Internal Benchmarks	196
7.4	Competitive Dynamics and Winning Coalitions	199
7.4.1	General Considerations	199
7.4.2	Associative Thought Processes	203
7.4.3	Autonomous Online Learning	207
7.5	Environmental Model Building	211
7.5.1	The Elman Simple Recurrent Network	211
7.5.2	Universal Prediction Tasks	215
	Exercises	218
	Further Reading	218
8	Solutions	221
	Index	241