GLOBAL EDITION

Engineering Mechanics
STATICS

Fifteenth Edition in SI Units

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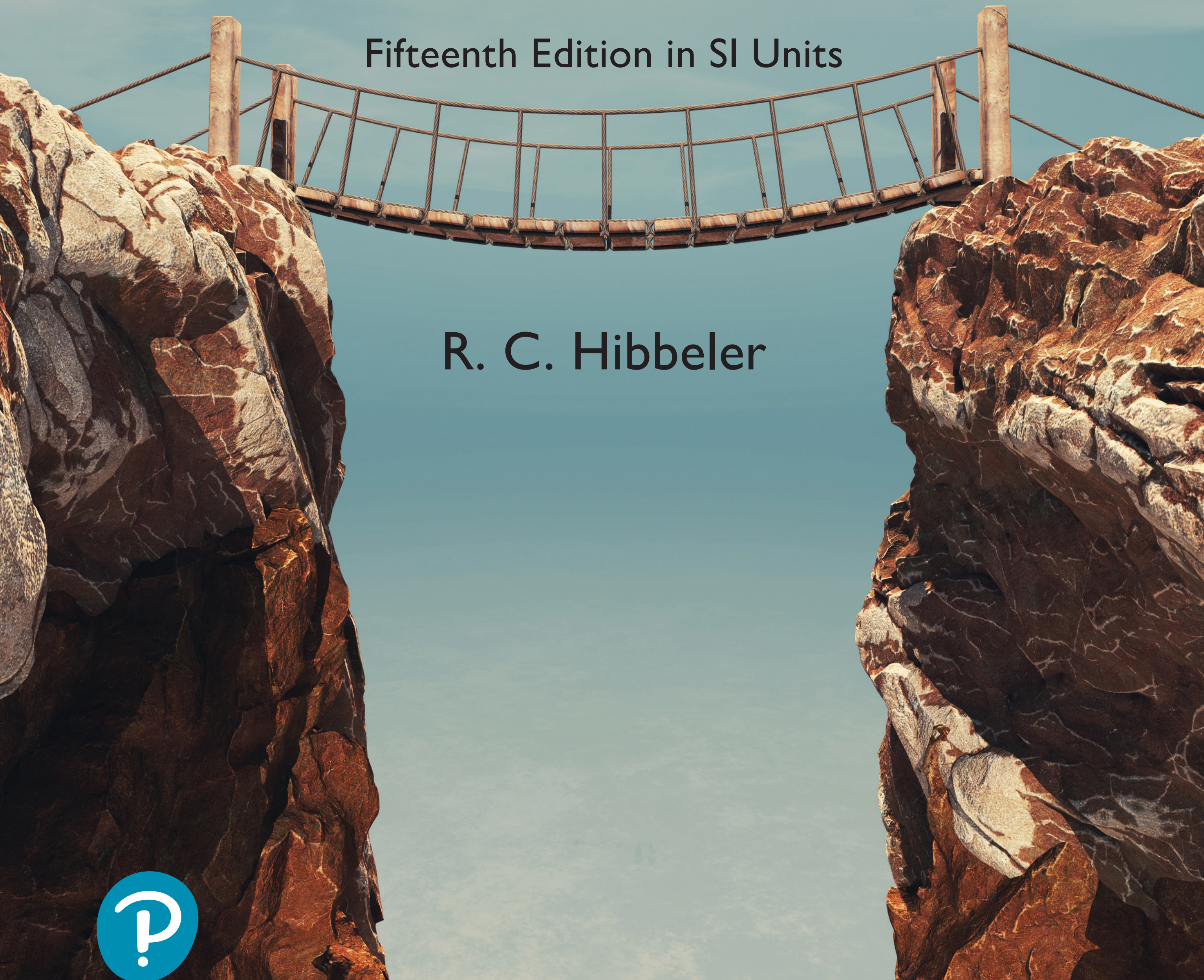


SI Prefixes

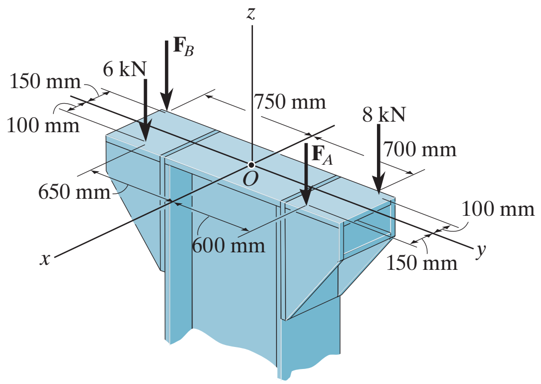
Multiple	*Exponential Form*	*Prefix*	*SI Symbol*
1 000 000 000	10^9	giga	G
1 000 000	10^6	mega	M
1 000	10^3	kilo	k
Submultiple			
0.001	10^{-3}	milli	m
0.000 001	10^{-6}	micro	μ
0.000 000 001	10^{-9}	nano	n

Conversion Factors (SI) to (FPS)

Quantity	*Unit of Measurement (SI)*	*Equals*	*Unit of Measurement (FPS)*
Force	N		0.2248 lb
Mass	kg		0.06852 slug
Length	m		3.281 ft

4–141. If $F_A = 7$ kN and $F_B = 5$ kN, represent the force system by a resultant force, and specify its location on the x–y plane.

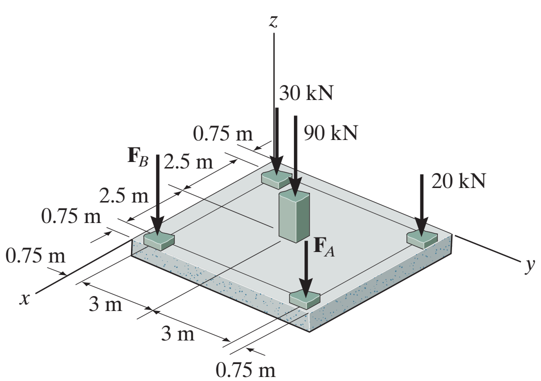
4–142. Determine the magnitudes of $\mathbf{F}_A$ and $\mathbf{F}_B$ so that the resultant force passes through point O.



Probs. 4–141/142

4–143. If $F_A = 40$ kN and $F_B = 35$ kN, determine the magnitude of the resultant force and specify the location of its point of application (x, y) on the slab.

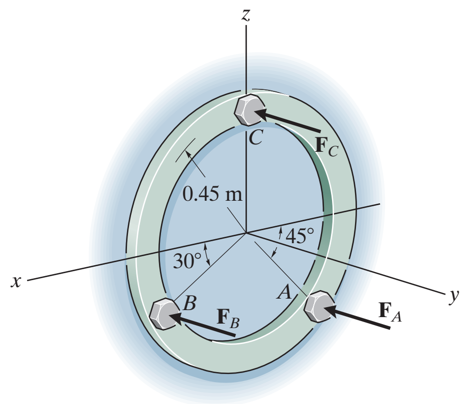
***4–144.** If the resultant force is required to act at the center of the slab, determine the magnitude of the column loadings $\mathbf{F}_A$ and $\mathbf{F}_B$ and the magnitude of the resultant force.



Probs. 4–143/144

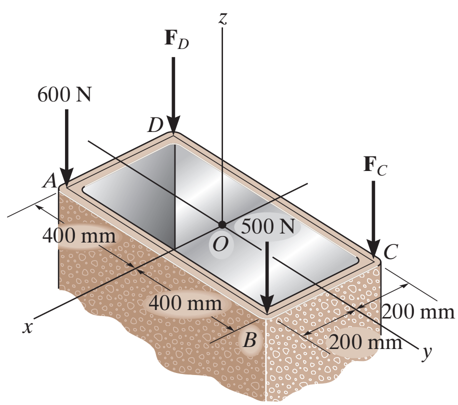
4–145. Three parallel bolting forces act on the circular plate. Determine the resultant force, and specify its location (x, z) on the plate. $F_A = 900$ N, $F_B = 450$ N, and $F_C = 1.80$ kN.

4–146. The three parallel bolting forces act on the circular plate. If the force at A has a magnitude of $F_A = 900$ N, determine the magnitudes of $\mathbf{F}_B$ and $\mathbf{F}_C$ so that the resultant force $\mathbf{F}_R$ of the system has a line of action that coincides with the y axis. *Hint:* This requires $\Sigma M_x = 0$ and $\Sigma M_z = 0$.



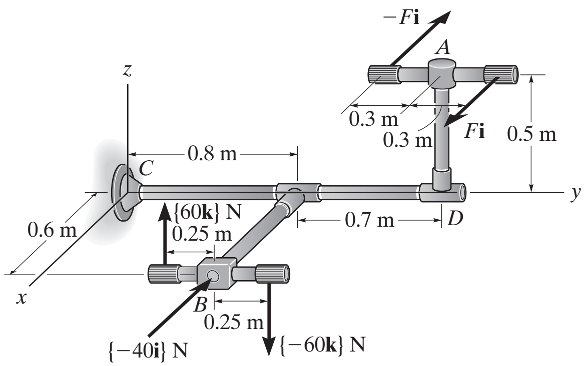
Probs. 4–145/146

4–147. The tube supports the four parallel forces. Determine the magnitudes of forces $\mathbf{F}_C$ and $\mathbf{F}_D$ acting at C and D so that the equivalent resultant force of the force system acts through the midpoint O of the tube.



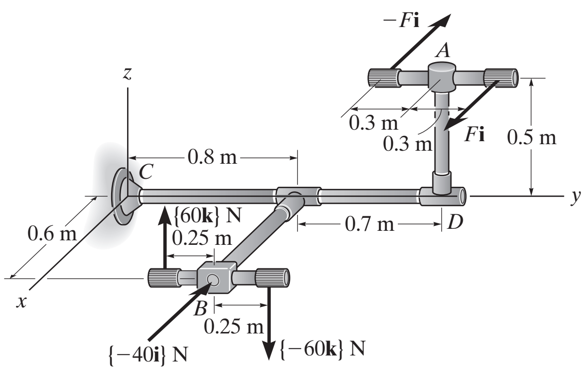
Prob. 4–147

***4–148.** The pipe assembly is subjected to the action of a wrench at B and a couple at A. Determine the magnitude F of the couple forces so that the system can be simplified to a wrench acting at point C.



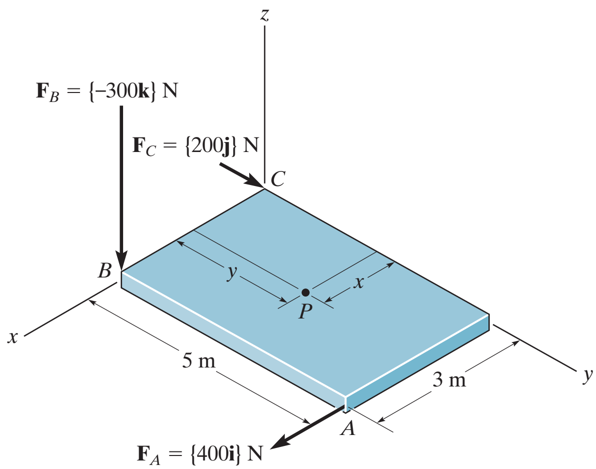
Prob. 4–148

4–149. The pipe assembly is subjected to the action of a wrench at B and a couple at A. Simplify this system to a resultant wrench and specify the location of the wrench along the axis of pipe CD, measured from point C. Set $F = 40$ N.



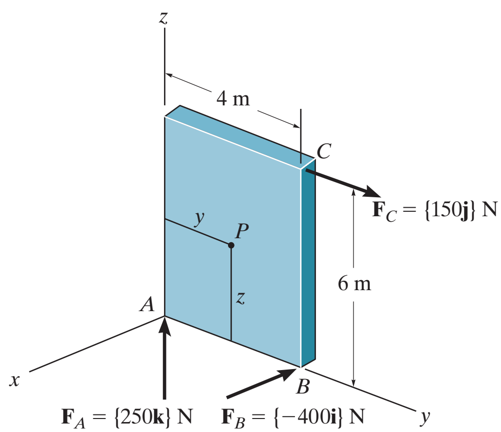
Prob. 4–149

4–150. Replace the three forces acting on the plate by a wrench. Specify the magnitude of the force and couple moment for the wrench and the point $P(x, y)$ where the wrench intersects the plate.



Prob. 4–150

4–151. Replace the three forces acting on the plate by a wrench. Specify the magnitude of the force and couple moment for the wrench and the point $P(y, z)$ where its line of action intersects the plate.



Prob. 4–151

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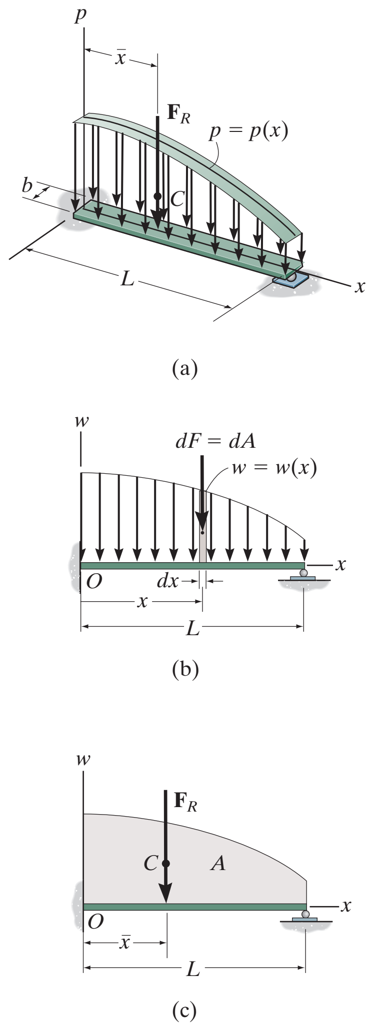


Fig. 4–48

4.9 REDUCTION OF A SIMPLE DISTRIBUTED LOADING

Sometimes, a body may be subjected to a loading that is distributed over its surface. For example, wind on the face of a sign, water within a tank, or the weight of sand on the floor of a storage container all exert ***distributed loadings***. The pressure caused by these loadings at each point on the surface represents the intensity of the loading. It is measured using pascals, Pa, (or N/m^2) in SI units.

Loading Along a Single Axis.

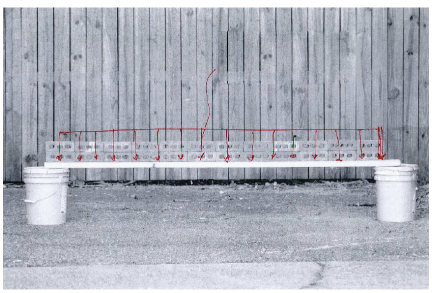
The most common type of distributed pressure loading is represented along a single axis.* For example, consider the beam (or plate) in Fig. 4–48*a* that has a constant width and is subjected to a pressure loading that varies only along the x axis. This loading can be described by the function $p = p(x)\ \text{N/m}^2$. Since it contains only one variable, x, we can represent it as a *coplanar distributed load*. To do so, we must multiply it by the width b m of the beam, so that $w(x) = p(x)b\ \text{N/m}$, Fig. 4–48*b*. Using the methods of Sec. 4.8, we can replace this coplanar parallel force system with a single equivalent resultant force $\mathbf{F}_R$, Fig. 4–48*c*.

Magnitude of Resultant Force.

The magnitude of $\mathbf{F}_R$ is equivalent to the sum of all the forces in the system, $F_R = \Sigma F$. In this case integration must be used since there is an infinite number of parallel forces $d\mathbf{F}$ acting on the beam, Fig. 4–48*b*. Each $d\mathbf{F}$ is acting on an element of length dx, and since $w(x)$ is a force per unit length, then $dF = w(x)\ dx = dA$. For the entire length L,

$$+\downarrow F_R = \Sigma F; \qquad \boxed{F_R = \int_L w(x)\ dx = \int_A dA = A} \qquad (4\text{–}19)$$

Therefore, the magnitude of the resultant force is equal to the area A under the loading diagram, Fig. 4–48*c*.



The pile of bricks creates a uniform distributed loading on the board.

*The more general case of a surface loading acting on a body is considered in Sec. 9.5.

Location of Resultant Force. The location $\bar{x}$ of $\mathbf{F}_R$ can be determined by equating the moments of the force resultant and the parallel force distribution about point O. Since $d\mathbf{F}$ produces a moment of $x\,dF = x\,[w(x)\,dx]$ about O, Fig. 4–48b, then for the entire length L, Fig. 4–48c,

$$\curvearrowright + (M_R)_O = \Sigma M_O; \qquad -\bar{x}F_R = -\int_L x w(x)\,dx$$

Solving for $\bar{x}$, using Eq. 4–19, we have

$$\boxed{\bar{x} = \frac{\displaystyle\int_L x w(x)\,dx}{\displaystyle\int_L w(x)\,dx} = \frac{\displaystyle\int_A x\,dA}{\displaystyle\int_A dA}} \qquad (4\text{–}20)$$

This coordinate $\bar{x}$ locates the geometric center or ***centroid*** of the *area* under the distributed loading. *In other words, the line of action of the resultant force passes through the centroid C (geometric center) of the area under the loading diagram*, Fig. 4–48c.

When the distributed-loading diagram is in the shape of a rectangle, triangle, or some other simple geometric form, then the centroid location for such common shapes does not have to be determined from the above equation. Rather it can be obtained directly from the tabulation given on the inside back cover.

Once $\bar{x}$ is determined, $\mathbf{F}_R$ by symmetry passes through point $(\bar{x}, 0)$ on the surface of the beam in Fig. 4–48a. *And so in three dimensions the resultant force has a magnitude equal to the volume under the loading curve $p = p(x)$ and a line of action which passes through the centroid (geometric center) of this volume.*

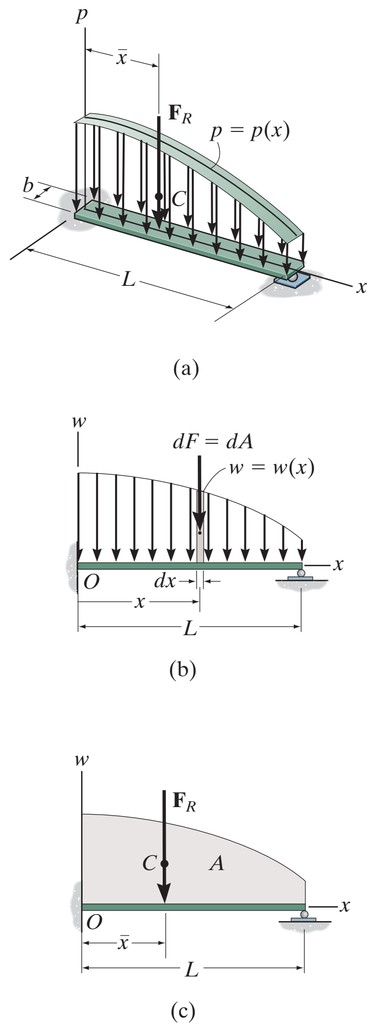


Fig. 4–48 (Repeated)

IMPORTANT POINTS

- Coplanar distributed loadings are defined by using a loading function $w = w(x)$ that indicates the intensity of the loading along the length of a member. This intensity is measured in N/m.
- The external effects caused by a coplanar distributed load acting on a member can be represented by a resultant force.
- This resultant is equivalent to the *area* under the loading diagram, and has a line of action that passes through the *centroid* or geometric center of this area.

Refer to the companion website for Lecture Summary and Quiz videos.

EXAMPLE 4.21

Determine the magnitude and location of the equivalent resultant force acting on the shaft in Fig. 4–49*a*.

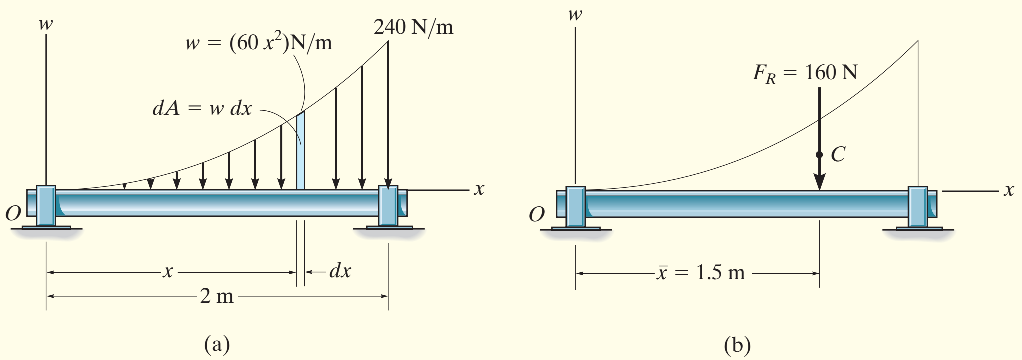


Fig. 4–49

SOLUTION

Since $w = w(x)$ is given, this problem will be solved by integration. The differential element has an area $dA = w\,dx = 60x^2\,dx$. Applying Eq. 4–19,

$$+\downarrow F_R = \Sigma F;$$

$$F_R = \int_A dA = \int_0^{2\text{ m}} 60x^2\,dx = 60\left(\frac{x^3}{3}\right)\Bigg|_0^{2\text{ m}} = 60\left(\frac{2^3}{3} - \frac{0^3}{3}\right)$$

$$= 160\text{ N} \qquad \textit{Ans.}$$

The location $\bar{x}$ of $\mathbf{F}_R$ measured from O, Fig. 4–49*b*, is determined from Eq. 4–20.

$$\bar{x} = \frac{\displaystyle\int_A x\,dA}{\displaystyle\int_A dA} = \frac{\displaystyle\int_0^{2\text{ m}} x(60x^2)\,dx}{160\text{ N}} = \frac{60\left(\dfrac{x^4}{4}\right)\Bigg|_0^{2\text{ m}}}{160\text{ N}} = \frac{60\left(\dfrac{2^4}{4} - \dfrac{0^4}{4}\right)}{160\text{ N}}$$

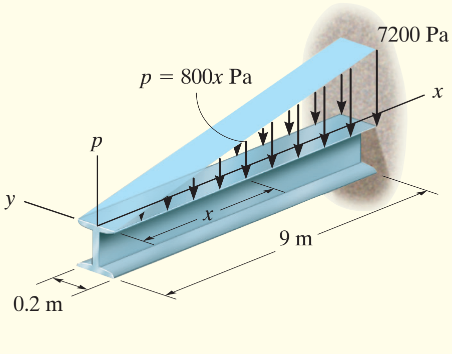
$$= 1.5\text{ m} \qquad \textit{Ans.}$$

NOTE: These results can be checked by using the table on the inside back cover, where it is shown that the formula for an exparabolic area of length a, height b, and shape shown in Fig. 4–49*a*, is

$$A = \frac{ab}{3} = \frac{2\text{ m}(240\text{ N/m})}{3} = 160\text{ N and } \bar{x} = \frac{3}{4}a = \frac{3}{4}(2\text{ m}) = 1.5\text{ m}$$

EXAMPLE 4.22

A distributed loading of $p = (800x)$ Pa acts over the top surface of the beam shown in Fig. 4–50*a*. Determine the magnitude and location of the equivalent resultant force.



(a)

SOLUTION

Since the loading intensity is uniform along the width of the beam (the y axis), the loading can be viewed in two dimensions as shown in Fig. 4–50*b*. Here

$$w = (800x\ \text{N/m}^2)(0.2\ \text{m})$$
$$= (160x)\ \text{N/m}$$

At $x = 9$ m, $w = 1440$ N/m. Although we may again apply Eqs. 4–19 and 4–20 as in the previous example, it is simpler to use the table on the inside back cover.

The magnitude of the resultant force is equivalent to the area of the triangle.

$$F_R = \tfrac{1}{2}(9\ \text{m})(1440\ \text{N/m}) = 6480\ \text{N} = 6.48\ \text{kN} \qquad \textit{Ans.}$$

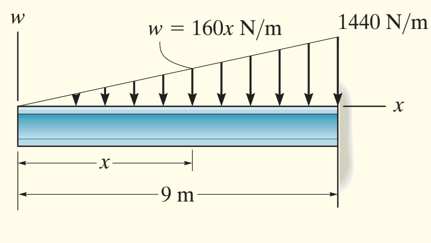
The line of action of $\mathbf{F}_R$ passes through the *centroid* C of this triangle. Hence,

$$\bar{x} = 9\ \text{m} - \tfrac{1}{3}(9\ \text{m}) = 6\ \text{m} \qquad \textit{Ans.}$$

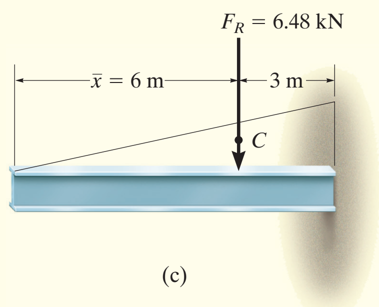
The results are shown in Fig. 4–50*c*.

NOTE: We may also view the resultant $\mathbf{F}_R$ as *acting* through the *centroid* of the *volume* of the loading diagram $p = p(x)$ in Fig. 4–50*a*. Then $\mathbf{F}_R$ intersects the x–y plane at the point (6 m, 0). Furthermore, the magnitude of $\mathbf{F}_R$ is equal to the volume under this loading diagram; i.e.,

$$F_R = V = \tfrac{1}{2}(7200\ \text{N/m}^2)(9\ \text{m})(0.2\ \text{m}) = 6.48\ \text{kN} \qquad \textit{Ans.}$$



(b)



(c)

Fig. 4–50

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