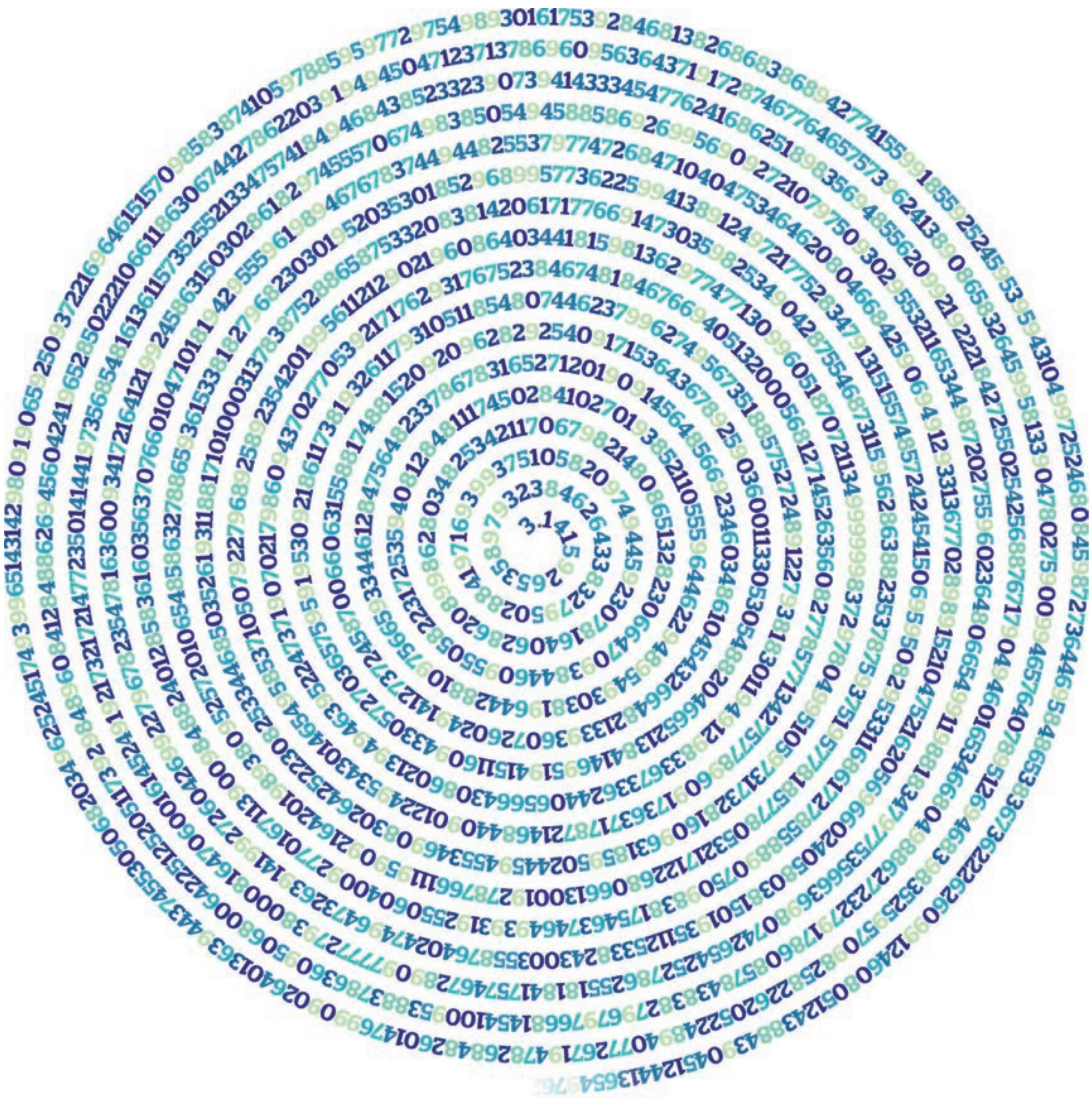
GLOBAL EDITION



Algebra and Trigonometry

SEVENTH EDITION

ROBERT F. BLITZER

A Brief Guide to Getting the Most from This Book

1 Read the Book

Feature	Description	Benefit
Section-Opening Scenarios	Every section opens with a scenario presenting a unique application of algebra or trigonometry in your life outside the classroom.	Realizing that algebra and trigonometry are everywhere will help motivate your learning. **(See page 136.)**
EXAMPLE	Examples are clearly written and provide step-by-step solutions. No steps are omitted, and each step is thoroughly explained to the right of the mathematics.	The blue annotations will help you understand the solutions by providing the reason why every algebraic or trigonometric step is true. **(See page 138.)**
Applications Using Real-World Data	Interesting applications from nearly every discipline, supported by up-to-date real-world data, are included in every section.	Ever wondered how you'll use algebra and trigonometry? This feature will show you how they can solve real problems. **(See pages 294–296.)**
GREAT QUESTION!	Answers to students' questions offer suggestions for problem solving, point out common errors to avoid, and provide informal hints and suggestions.	By seeing common mistakes, you'll be able to avoid them. This feature should help you not to feel anxious or threatened when asking questions in class. **(See page 139.)**
BRIEF REVIEW	Brief Reviews cover skills you already learned but may have forgotten.	Having these refresher boxes easily accessible will help ease anxiety about skills you may have forgotten. **(See page 508.)**
BLITZER BONUS	These enrichment essays provide historical, interdisciplinary, and otherwise interesting connections to the algebra or trigonometry under study.	Yet even more proof that math is an interesting and dynamic discipline! **(See page 195.)**
Explanatory Voice Balloons	Voice balloons help to demystify algebra and trigonometry. They translate math into plain English, clarify problem-solving procedures, and present alternative ways of understanding.	Does math ever look foreign to you? This feature often translates math into everyday English. **(See page 193.)**
WHAT YOU'LL LEARN 1 Learning Objective	Every section begins with a list of objectives. Each objective is restated in the margin where the objective is covered.	The objectives focus your reading by emphasizing what is most important and where to find it. **(See page 178.)**
TECHNOLOGY	The screens displayed in the technology boxes show how graphing utilities verify and visualize algebraic or trigonometric results.	Even if you are not using a graphing utility in the course, this feature will help you understand different approaches to problem solving. **(See page 140.)**

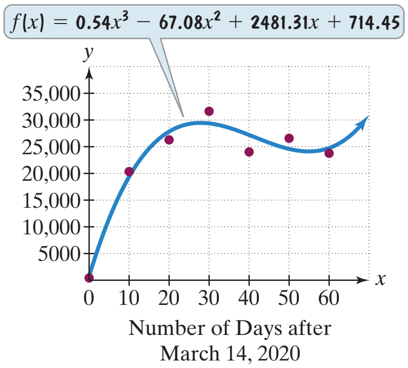


Figure 3.12 The graph of a function modeling the new coronavirus cases shown in **Figure 3.11**, with the actual data as a set of seven points

The function

$$f(x) = 0.54x^3 - 67.08x^2 + 2481.31x + 714.45$$

models the number of new coronavirus cases, $f(x)$, x days after March 14, 2020. **Figure 3.12** presents the data in **Figure 3.11** as a set of seven points and shows the graph of f passing near the data points. This function is an example of a *polynomial function of degree 3*.

Definition of a Polynomial Function

Let n be a nonnegative integer and let $a_n, a_{n-1}, \ldots, a_2, a_1, a_0$ be real numbers, with $a_n \neq 0$. The function defined by

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2x^2 + a_1x + a_0$$

is called a **polynomial function of degree n**. The number a_n, the coefficient of the variable to the highest power, is called the **leading coefficient**.

GREAT QUESTION!

What kind of notation is $a_n, a_{n-1}, a_2, a_1, a_0$? I find it confusing.

When defining polynomial functions, we use *subscripted letters*. **Subscripts** are written next to, but slightly below, each a.

- a_n ("a sub-n"): the coefficient of x^n
- a_{n-1} ("a sub-$n-1$"): the coefficient of x^{n-1}
- a_2 ("a sub-2"): the coefficient of x^2
- a_1 ("a sub-1"): the coefficient of x
- a_0 ("a sub-0"): the constant term

Polynomial Functions	Not Polynomial Functions
$f(x) = -3x^5 + \sqrt{2}x^2 + 5$ Polynomial function of degree 5	$F(x) = -3\sqrt{x} + \sqrt{2}x^2 + 5$ $= -3x^{\frac{1}{2}} + \sqrt{2}x^2 + 5$ The exponent on the variable is not an integer.
$g(x) = -3x^4(x-2)(x+3)$ $= -3x^4(x^2 + x - 6)$ $= -3x^6 - 3x^5 + 18x^4$ Polynomial function of degree 6	$G(x) = -\frac{3}{x^2} + \sqrt{2}x^2 + 5$ $= -3x^{-2} + \sqrt{2}x^2 + 5$ The exponent on the variable is not a nonnegative integer.

A constant function $f(x) = c$, where $c \neq 0$, is a polynomial function of degree 0. A linear function $f(x) = mx + b$, where $m \neq 0$, is a polynomial function of degree 1. A quadratic function $f(x) = ax^2 + bx + c$, where $a \neq 0$, is a polynomial function of degree 2. In this section, we focus on polynomial functions of degree 3 or higher.

2 Recognize characteristics of graphs of polynomial functions.

Smooth, Continuous Graphs

Polynomial functions of degree 2 or higher have graphs that are *smooth* and *continuous*. By **smooth**, we mean that the graphs contain only rounded curves with no sharp corners. By **continuous**, we mean that the graphs have no breaks and can be drawn without lifting your pencil from the rectangular coordinate system. These ideas are illustrated in **Figure 3.13**.

Graphs of Polynomial Functions | **Not Graphs of Polynomial Functions**

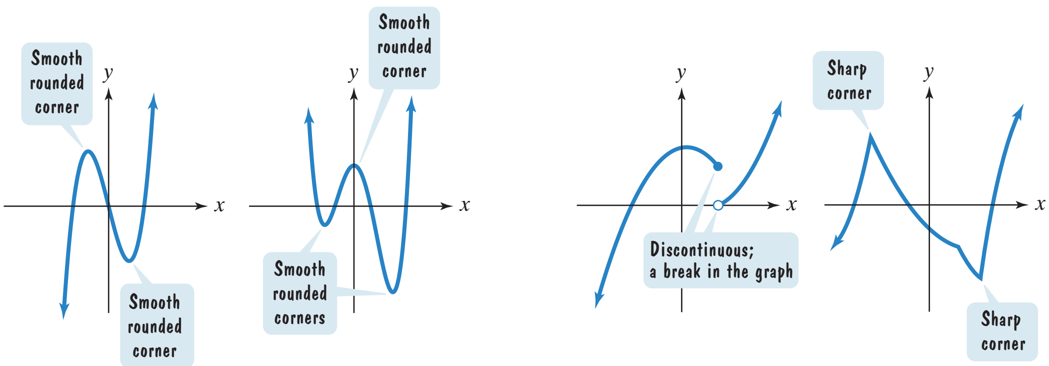


Figure 3.13 Recognizing graphs of polynomial functions

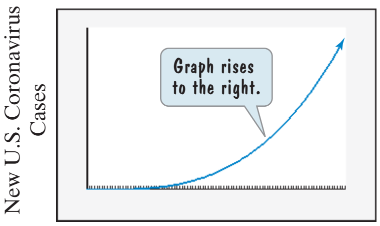
3 Determine end behavior.

End Behavior of Polynomial Functions

Figure 3.14 shows the graph of the function

$$f(x) = 0.54x^3 - 67.08x^2 + 2481.31x + 714.45,$$

which models the number of new U.S. coronavirus cases, $f(x)$, x days after March 14, 2020. Look at what happens to the graph when we extend the number of days past March 14 to 1000. By 1000 days past March 14 (approximately 2.7 years), the value of y exceeds 400 million, which is more than the U.S. population. We've added an arrow to the graph at the far right to emphasize that it continues to increase without bound. It is this far-right *end behavior* of the graph that makes it inappropriate for modeling the number of new U.S. coronavirus cases into the future.



[0, 1000, 10] by [0, 500,000,000, 5000]

Figure 3.14 By extending the viewing rectangle to 1000, y exceeds the U.S. population and the function no longer models the number of new COVID-19 virus cases.

The behavior of the graph of a function to the far left or the far right is called its **end behavior**. Although the graph of a polynomial function may have intervals where it increases or decreases, the graph will eventually rise or fall without bound as it moves far to the left or far to the right.

How can you determine whether the graph of a polynomial function goes up or down at each end? The end behavior of a polynomial function

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$$

depends upon the leading term $a_n x^n$ because when $|x|$ is large, the other terms are relatively insignificant in size. In particular, the sign of the leading coefficient, a_n, and the degree, n, of the polynomial function reveal its end behavior. In terms of end behavior, only the term of highest degree counts, as summarized by the **Leading Coefficient Test**.

The Leading Coefficient Test

As x increases or decreases without bound, the graph of the polynomial function

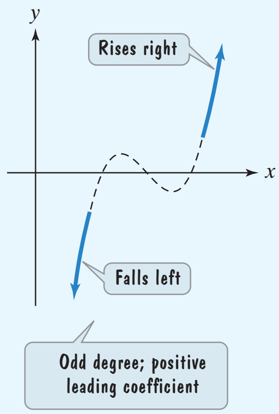
$$f(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_1 x + a_0 \quad (a_n \neq 0)$$

eventually rises or falls. In particular,

1. For n odd:

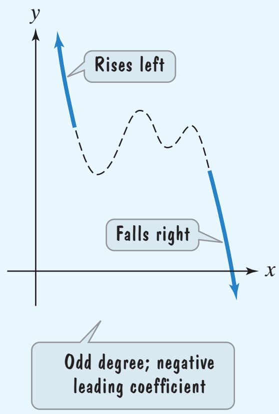
If the leading coefficient is positive, the graph falls to the left and rises to the right. (↙, ↗)

$a_n > 0$



If the leading coefficient is negative, the graph rises to the left and falls to the right. (↖, ↘)

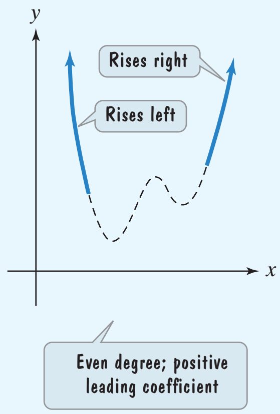
$a_n < 0$



2. For n even:

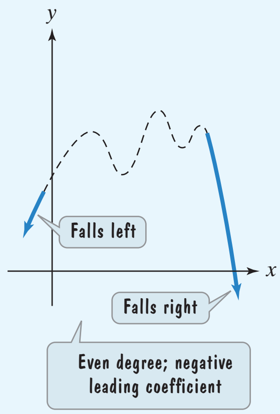
If the leading coefficient is positive, the graph rises to the left and rises to the right. (↖, ↗)

$a_n > 0$



If the leading coefficient is negative, the graph falls to the left and falls to the right. (↙, ↘)

$a_n < 0$



DISCOVERY

Verify each of the four cases of the Leading Coefficient Test by using a graphing utility to graph $f(x) = x^3$, $f(x) = -x^3$, $f(x) = x^2$, and $f(x) = -x^2$.

GREAT QUESTION!

What's the bottom line on the Leading Coefficient Test?

Odd-degree polynomial functions have graphs with opposite behavior at each end. Even-degree polynomial functions have graphs with the same behavior at each end. Here's a table to help you remember the details:

Leading Term: $a_n x^n$

	Odd n	Even n
$a_n > 0$	↙ ↗	↖ ↗
$a_n < 0$	↖ ↘	↙ ↘

Opposite behavior at each end (Odd n) — Same behavior at each end (Even n)

EXAMPLE 1 Using the Leading Coefficient Test

Use the Leading Coefficient Test to determine the end behavior of the graph of

$$f(x) = x^3 + 3x^2 - x - 3.$$

Solution We begin by identifying the sign of the leading coefficient and the degree of the polynomial.

$$f(x) = x^3 + 3x^2 - x - 3$$

The leading coefficient, 1, is positive. The degree of the polynomial, 3, is odd.

The degree of the function f is 3, which is odd. Odd-degree polynomial functions have graphs with opposite behavior at each end. The leading coefficient, 1, is positive. Thus, the graph falls to the left and rises to the right (↙, ↗). The graph of f is shown in **Figure 3.15**.

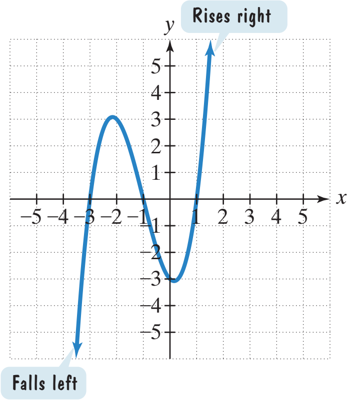


Figure 3.15 The graph of $f(x) = x^3 + 3x^2 - x - 3$

CHECK POINT 1 Use the Leading Coefficient Test to determine the end behavior of the graph of $f(x) = x^4 - 4x^2$.

EXAMPLE 2 Using the Leading Coefficient Test

Use the Leading Coefficient Test to determine the end behavior of the graph of

$$f(x) = -4x^3(x - 1)^2(x + 5).$$

Solution Although the equation for f is in factored form, it is not necessary to multiply to determine the degree of the function.

$$f(x) = -4x^3(x - 1)^2(x + 5)$$

Degree of this factor is 3. Degree of this factor is 2. Degree of this factor is 1.

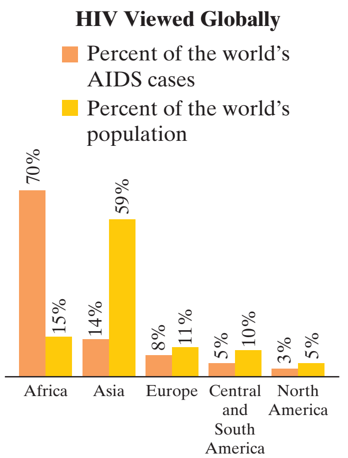
When multiplying exponential expressions with the same base, we add the exponents. This means that the degree of f is $3 + 2 + 1$, or 6, which is even. Even-degree polynomial functions have graphs with the same behavior at each end. Without multiplying out, you can see that the leading coefficient is -4, which is negative. Thus, the graph of f falls to the left and falls to the right (↙, ↘).

CHECK POINT 2 Use the Leading Coefficient Test to determine the end behavior of the graph of $f(x) = 2x^3(x - 1)(x + 5)$.

BLITZER BONUS

AIDS: A Global Perspective

Unlike the coronavirus, by the end of 2020 the AIDS virus had been brought under control. For the over one million Americans who received treatment, HIV had become a chronic disease, a condition they live with. Globally, by contrast, with 21 million new HIV infections a year, nearly 6000 every day, the epidemic continues to explode. Overall, between 35 to 40 million people have died from AIDS.



Source: Based on James M. Henslin, *Sociology*, 14th edition, Pearson, 2019

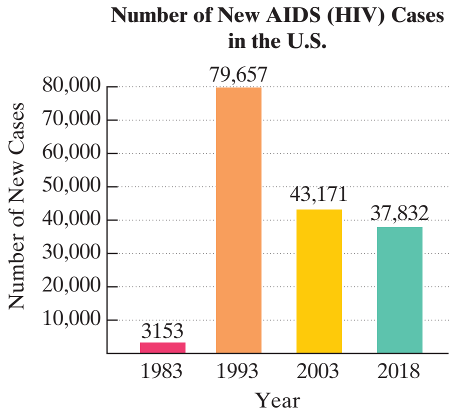
EXAMPLE 3 Using the Leading Coefficient Test

AIDS (HIV) remains a persistent problem for the United States and countries around the world. According to the CDC (Centers for Disease Control and Prevention), at the end of 2020, an estimated 1.2 million people in the United States had AIDS.

Using a limited portion of the data shown by the bar graph, we developed the model

$$f(x) = -49x^3 + 806x^2 + 3776x + 2503$$

for the number of Americans with AIDS, $f(x)$, x years after 1983. Use end behavior to explain why this model is only appropriate for a limited time period.



Solution We begin by identifying the sign of the leading coefficient and the degree of the polynomial.

$$f(x) = -49x^3 + 806x^2 + 3776x + 2503$$

The leading coefficient, −49, is negative.

The degree of the polynomial, 3, is odd.

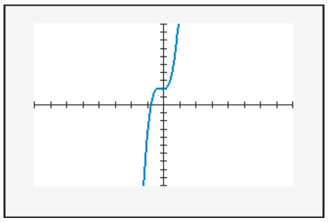
The degree of f is 3, which is odd. Odd-degree polynomial functions have graphs with opposite behavior at each end. The leading coefficient, −49, is negative. Thus, the graph rises to the left and falls to the right (↖, ↘). The fact that the graph falls to the right indicates that at some point the number of AIDS cases will be negative, an impossibility. If a function has a graph that decreases without bound over time, it will not be capable of modeling nonnegative phenomena over long time periods. Model breakdown will eventually occur.

CHECK POINT 3 The polynomial function

$$f(x) = -0.27x^3 + 9.2x^2 - 102.9x + 400$$

models the ratio of students to computers in U.S. public schools x years after 1980. Use end behavior to determine whether this function could be an appropriate model for computers in the classroom well into the twenty-first century. Explain your answer.

If you use a graphing utility to graph a polynomial function, it is important to select a viewing rectangle that accurately reveals the graph's end behavior. If the viewing rectangle, or window, is too small, it may not accurately show a complete graph with the appropriate end behavior.



[−8, 8, 1] by [−10, 10, 1]

Figure 3.16

EXAMPLE 4 Using the Leading Coefficient Test

The graph of $f(x) = -x^4 + 8x^3 + 4x^2 + 2$ was obtained with a graphing utility using a [−8, 8, 1] by [−10, 10, 1] viewing rectangle. The graph is shown in **Figure 3.16**. Is this a complete graph that shows the end behavior of the function?

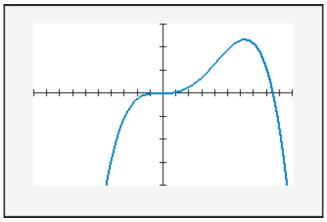
Solution We begin by identifying the sign of the leading coefficient and the degree of the polynomial.

$$f(x) = -x^4 + 8x^3 + 4x^2 + 2$$

The leading coefficient, −1, is negative.

The degree of the polynomial, 4, is even.

The degree of f is 4, which is even. Even-degree polynomial functions have graphs with the same behavior at each end. The leading coefficient, −1, is negative. Thus, the graph should fall to the left and fall to the right (↙, ↘). The graph in **Figure 3.16** is falling to the left, but it is not falling to the right. Therefore, the graph is not complete enough to show end behavior. A more complete graph of the function is shown in a larger viewing rectangle in **Figure 3.17**.



[−10, 10, 1] by [−1000, 750, 250]

Figure 3.17

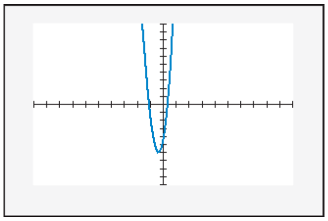


Figure 3.18

CHECK POINT 4 The graph of $f(x) = x^3 + 13x^2 + 10x - 4$ is shown in a standard viewing rectangle in **Figure 3.18**. Use the Leading Coefficient Test to determine whether this is a complete graph that shows the end behavior of the function. Explain your answer.

4 Use factoring to find zeros of polynomial functions.

Zeros of Polynomial Functions

If f is a polynomial function, then the values of x for which $f(x)$ is equal to 0 are called the **zeros** of f. These values of x are the **roots**, or **solutions**, of the polynomial equation $f(x) = 0$. Each real root of the polynomial equation appears as an x-intercept of the graph of the polynomial function.

EXAMPLE 5 Finding Zeros of a Polynomial Function

Find all zeros of $f(x) = x^3 + 3x^2 - x - 3$.

Solution By definition, the zeros are the values of x for which $f(x)$ is equal to 0. Thus, we set $f(x)$ equal to 0:

$$f(x) = x^3 + 3x^2 - x - 3 = 0.$$

We solve the polynomial equation $x^3 + 3x^2 - x - 3 = 0$ for x as follows:

$$x^3 + 3x^2 - x - 3 = 0$$ This is the equation needed to find the function's zeros.

$$x^2(x + 3) - 1(x + 3) = 0$$ Factor x^2 from the first two terms and −1 from the last two terms.

$$(x + 3)(x^2 - 1) = 0$$ A common factor of $x + 3$ is factored from the expression.

$$x + 3 = 0 \quad \text{or} \quad x^2 - 1 = 0$$ Set each factor equal to 0.

$$x = -3 \qquad x^2 = 1$$ Solve for x.

$$x = \pm 1$$ Remember that if $x^2 = d$, then $x = \pm\sqrt{d}$.

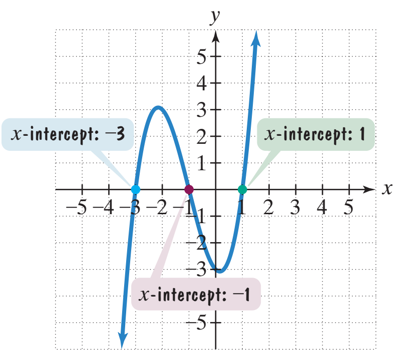


Figure 3.19

The zeros of f are −3, −1, and 1. The graph of f in **Figure 3.19** shows that each zero is an x-intercept. The graph passes through the points (−3, 0), (−1, 0), and (1, 0).

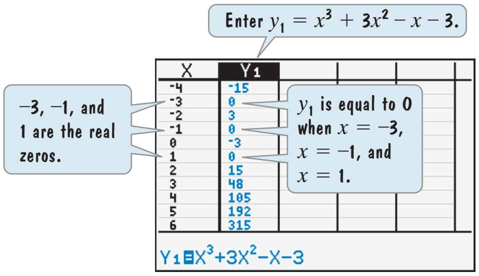
TECHNOLOGY

Graphic and Numeric Connections

A graphing utility can be used to verify that -3, -1, and 1 are the three real zeros of $f(x) = x^3 + 3x^2 - x - 3$.

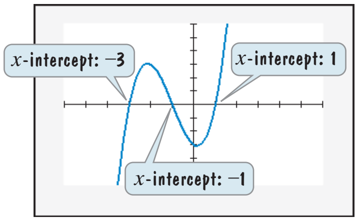
Numeric Check

Display a table for the function.



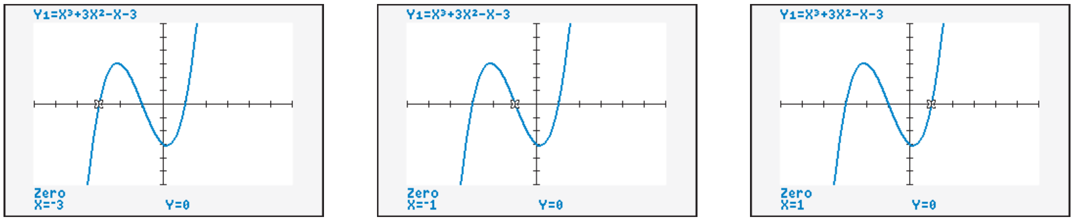
Graphic Check

Display a graph for the function. The x-intercepts indicate that -3, -1, and 1 are the real zeros.



$[-6, 6, 1]$ by $[-6, 6, 1]$

The utility's ZERO feature on the graph of f also verifies that -3, -1, and 1 are the function's real zeros.



CHECK POINT 5 Find all zeros of $f(x) = x^3 + 2x^2 - 4x - 8$.

EXAMPLE 6 Finding Zeros of a Polynomial Function

Find all zeros of $f(x) = -x^4 + 4x^3 - 4x^2$.

Solution We find the zeros of f by setting $f(x)$ equal to 0 and solving the resulting equation.

$$-x^4 + 4x^3 - 4x^2 = 0 \quad \text{We now have a polynomial equation.}$$
$$x^4 - 4x^3 + 4x^2 = 0 \quad \text{Multiply both sides by } -1. \text{ This step is optional.}$$
$$x^2(x^2 - 4x + 4) = 0 \quad \text{Factor out } x^2.$$
$$x^2(x - 2)^2 = 0 \quad \text{Factor completely.}$$
$$x^2 = 0 \quad \text{or} \quad (x - 2)^2 = 0 \quad \text{Set each factor equal to 0.}$$
$$x = 0 \qquad x = 2 \quad \text{Solve for } x.$$

The zeros of $f(x) = -x^4 + 4x^3 - 4x^2$ are 0 and 2. The graph of f, shown in **Figure 3.20**, has x-intercepts at 0 and 2. The graph passes through the points $(0, 0)$ and $(2, 0)$.

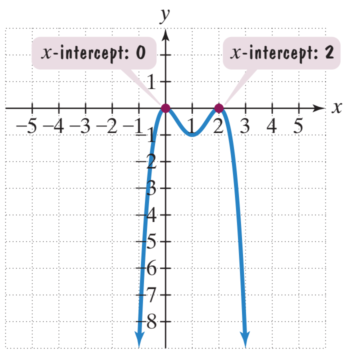


Figure 3.20 The zeros of $f(x) = -x^4 + 4x^3 - 4x^2$, namely, 0 and 2, are the x-intercepts for the graph of f.

CHECK POINT 6 Find all zeros of $f(x) = x^4 - 4x^2$.