GLOBAL EDITION

Physics for Scientists and Engineers

A Strategic Approach with Modern Physics

FIFTH EDITION

Randall D. Knight

PHYSICS

For Scientists and Engineers | A Strategic Approach

WITH MODERN PHYSICS

GLOBAL EDITION

The A_1 and Δx_1 enter the equation from different terms, but they conveniently combine to give $\Delta V = A_1\,\Delta x_1$, the volume "vacated" by the shaded fluid as it's pushed forward.

The situation is much the same on the right edge of the system, where the surrounding fluid exerts a pressure force $\vec{F}_2 = (p_2 A_2,\ \text{to the left})$. However, force $\vec{F}_2$ is *opposite* the displacement $\Delta\vec{r}_2$, which introduces a minus sign into the dot product for the work, giving

$$W_2 = \vec{F}_2 \cdot \Delta\vec{r}_2 = -F_2\,\Delta x_2 = -(p_2 A_2)\Delta x_2 = -p_2(A_2\,\Delta x_2) = -p_2\,\Delta V \qquad (14.22)$$

Because the fluid is incompressible, the volume $\Delta V = A_2\,\Delta x_2$ "gained" on the right side is exactly the same as that lost on the left. Altogether, the work done on the system by the surrounding fluid is

$$W_{\text{ext}} = W_1 + W_2 = (p_1 - p_2)\Delta V \qquad (14.23)$$

The work depends on the *pressure difference* $p_1 - p_2$.

Now let's see what happens to the system's potential and kinetic energy. Most of the system does not change during time interval Δt; it's fluid at the same height moving at the same speed. All we need to consider are the volumes ΔV at the two ends. On the right, the system *gains* kinetic and gravitational potential energy as the fluid moves into volume ΔV. Simultaneously, the system *loses* kinetic and gravitational potential energy on the left as fluid vacates volume ΔV.

The mass of fluid in volume ΔV is $m = \rho\,\Delta V$, where ρ is the fluid density. Thus the net change in the system's gravitational potential energy during Δt is

$$\Delta U_{\text{G}} = mgy_2 - mgy_1 = \rho\Delta Vgy_2 - \rho\Delta Vgy_1 \qquad (14.24)$$

Similarly, the system's change in kinetic energy is

$$\Delta K = \tfrac{1}{2}mv_2^{\,2} - \tfrac{1}{2}mv_1^{\,2} = \tfrac{1}{2}\rho\Delta Vv_2^{\,2} - \tfrac{1}{2}\rho\Delta Vv_1^{\,2} \qquad (14.25)$$

Combining Equations 14.23, 14.24, and 14.25 gives us the energy equation for the fluid in the flow tube:

$$\tfrac{1}{2}\rho\Delta Vv_2^{\,2} - \tfrac{1}{2}\rho\Delta Vv_1^{\,2} + \rho\Delta Vgy_2 - \rho\Delta Vgy_1 = p_1\Delta V - p_2\Delta V \qquad (14.26)$$

The volume ΔV cancels out of all the terms. Regrouping terms, we have

$$p_1 + \tfrac{1}{2}\rho v_1^{\,2} + \rho g y_1 = p_2 + \tfrac{1}{2}\rho v_2^{\,2} + \rho g y_2 \qquad (14.27)$$

Equation 14.27 is called **Bernoulli's equation.** It is named for the 18th-century Swiss scientist Daniel Bernoulli, who made some of the earliest studies of fluid dynamics.

Bernoulli's equation is really nothing more than a statement about work and energy. It is sometimes useful to express Bernoulli's equation in the alternative form

$$p + \tfrac{1}{2}\rho v^2 + \rho g y = \text{constant} \qquad (14.28)$$

This version of Bernoulli's equation tells us that the quantity $p + \tfrac{1}{2}\rho v^2 + \rho g y$ remains constant along a streamline.

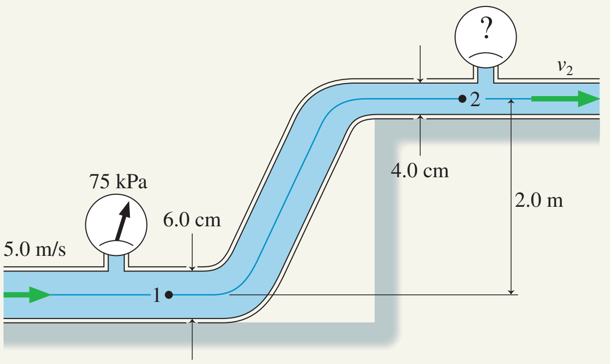
Much of modern industry is based on transporting fluids, both liquids and gases, from one point to another. The industrial sector of the economy consumes about one-third of all the energy produced in the United States, and a large fraction of that is used by pumps. Proper design—pipe sizes, lengths, and turns—is essential for the efficient use of energy.

NOTE Using Bernoulli's equation is very much like using the law of conservation of energy. Rather than identifying a "before" and "after," you want to identify two points on a streamline. As the following examples show, Bernoulli's equation is often used in conjunction with the equation of continuity.

EXAMPLE 14.11 ■ An irrigation system

Water flows through the pipes shown in **FIGURE 14.29**. The water's speed through the lower pipe is 5.0 m/s and a pressure gauge reads 75 kPa. What is the reading of the pressure gauge on the upper pipe?

FIGURE 14.29 The water pipes of an irrigation system.



MODEL Treat the water as an ideal fluid obeying Bernoulli's equation. Consider a streamline connecting point 1 in the lower pipe with point 2 in the upper pipe.

SOLVE Bernoulli's equation, Equation 14.27, relates the pressure, fluid speed, and heights at points 1 and 2. It is easily solved for the pressure p_2 at point 2:

$$p_2 = p_1 + \tfrac{1}{2}\rho v_1^2 - \tfrac{1}{2}\rho v_2^2 + \rho g y_1 - \rho g y_2$$
$$= p_1 + \tfrac{1}{2}\rho(v_1^2 - v_2^2) + \rho g(y_1 - y_2)$$

All quantities on the right are known except v_2, and that is where the equation of continuity will be useful. The cross-section areas and water speeds at points 1 and 2 are related by

$$v_1 A_1 = v_2 A_2$$

from which we find

$$v_2 = \frac{A_1}{A_2}v_1 = \frac{r_1^2}{r_2^2}v_1 = \frac{(0.030\text{ m})^2}{(0.020\text{ m})^2}(5.0\text{ m/s}) = 11.25\text{ m/s}$$

The pressure at point 1 is $p_1 = 75\text{ kPa} + 1\text{ atm} = 176{,}300\text{ Pa}$. We can now use the above expression for p_2 to calculate $p_2 = 105{,}900\text{ Pa}$. This is the absolute pressure; the pressure gauge on the upper pipe will read

$$p_2 = 105{,}900\text{ Pa} - 1\text{ atm} = 4.6\text{ kPa}$$

REVIEW Reducing the pipe size decreases the pressure because it makes $v_2 > v_1$. Gaining elevation also reduces the pressure.

EXAMPLE 14.12 ■ Hydroelectric power

Small hydroelectric plants in the mountains sometimes bring the water from a reservoir down to the power plant through enclosed tubes. In one such plant, the 100-cm-diameter intake tube in the base of the dam is 50 m below the reservoir surface. The water drops 200 m through the tube before flowing into the turbine through a 50-cm-diameter nozzle.

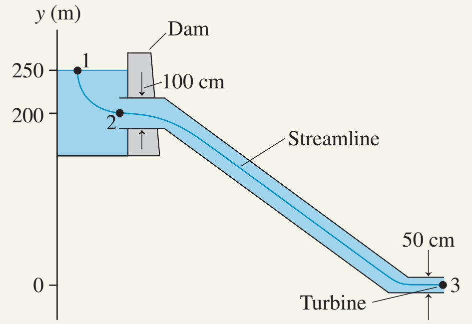
a. What is the water speed into the turbine?

b. By how much does the inlet pressure differ from the hydrostatic pressure at that depth?

MODEL Treat the water as an ideal fluid obeying Bernoulli's equation. Consider a streamline that begins at the surface of the reservoir and ends at the exit of the nozzle. The pressure at the surface is $p_1 = p_{atmos}$ and $v_1 \approx 0$ m/s. The water discharges into air, so $p_3 = p_{atmos}$ at the exit.

VISUALIZE **FIGURE 14.30** is a pictorial representation of the situation.

FIGURE 14.30 Pictorial representation of the water flow to a hydroelectric plant.



SOLVE a. Bernoulli's equation, with $v_1 = 0$ m/s and $y_3 = 0$ m, is

$$p_{\text{atmos}} + \rho g y_1 = p_{\text{atmos}} + \tfrac{1}{2}\rho v_3^2$$

The power plant is in the mountains, where $p_{\text{atmos}} < 1$ atm, but p_{atmos} occurs on both sides of Bernoulli's equation and cancels. Solving for v_3 gives

$$v_3 = \sqrt{2gy_1} = \sqrt{2(9.80\ \text{m/s}^2)(250\ \text{m})} = 70\ \text{m/s}$$

b. You might expect the pressure p_2 at the intake to be the hydrostatic pressure $p_{\text{atmos}} + \rho g d$ at depth d. But the water is *flowing* into the intake tube, so it's not in hydrostatic equilibrium. We can find the intake speed v_2 from the equation of continuity:

$$v_2 = \frac{A_3}{A_2}v_3 = \frac{r_3^2}{r_2^2}\sqrt{2gy_1}$$

The intake is along the streamline between points 1 and 3, so we can apply Bernoulli's equation to points 1 and 2:

$$p_{\text{atmos}} + \rho g y_1 = p_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2$$

Solving this equation for p_2, and noting that $y_1 - y_2 = d$, we find

$$\begin{aligned} p_2 &= p_{\text{atmos}} + \rho g(y_1 - y_2) - \tfrac{1}{2}\rho v_2^2 \\ &= p_{\text{atmos}} + \rho g d - \tfrac{1}{2}\rho\left(\frac{r_3}{r_2}\right)^4(2gy_1) \\ &= p_{\text{hydrostatic}} - \rho g y_1\left(\frac{r_3}{r_2}\right)^4 \end{aligned}$$

The intake pressure is *less* than hydrostatic pressure by the amount

$$\rho g y_1\left(\frac{r_3}{r_2}\right)^4 = 153{,}000\ \text{Pa} = 1.5\ \text{atm}$$

REVIEW The water's exit speed from the nozzle is the same as if it fell 250 m from the surface of the reservoir. This isn't surprising because we've assumed a nonviscous (i.e., frictionless) liquid. "Real" water would have less speed but still flow very fast.

NOTE Real gases are compressible. Real liquids have viscosity. It might seem that Bernoulli's equation, which assumes a nonviscous, incompressible fluid, would be a poor description of the real world, but it turns out that gases undergo little compression during most flows. And the viscosity of a "thin" liquid, such as water, is not a major issue unless a tube is very narrow. All in all, Bernoulli's equation is a reasonably good description of many real fluids.

Two Applications

The speed of a flowing gas is often measured with a device called a **Venturi tube.** Venturi tubes measure gas speeds in environments as different as chemistry laboratories, wind tunnels, and jet engines.

FIGURE 14.31 shows gas flowing through a tube that changes from cross-section area A_1 to area A_2. A U-shaped glass tube containing liquid of density ρ_{liq} connects the two segments of the flow tube. When gas flows through the horizontal tube, the liquid stands height h higher in the side of the U tube connected to the narrow segment of the flow tube.

Figure 14.31 shows how a Venturi tube works. We can make this analysis quantitative and determine the gas-flow speed from the liquid height h. Two pieces of information we have to work with are Bernoulli's equation

$$p_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 = p_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2 \tag{14.29}$$

and the equation of continuity

$$v_2 A_2 = v_1 A_1 \tag{14.30}$$

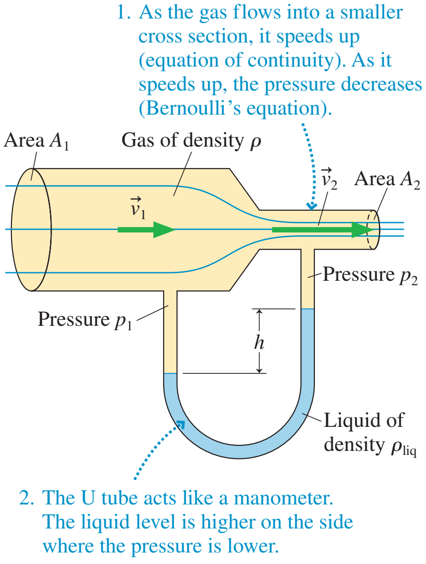
In addition, the hydrostatic equation for the liquid tells us that the pressure p_2 above the right tube differs from the pressure p_1 above the left tube by $\rho_{\text{liq}} g h$. That is,

$$p_2 = p_1 - \rho_{\text{liq}} g h \tag{14.31}$$

First we use Equations 14.30 and 14.31 to eliminate v_2 and p_2 in Bernoulli's equation:

$$p_1 + \tfrac{1}{2}\rho v_1^2 = (p_1 - \rho_{\text{liq}} g h) + \tfrac{1}{2}\rho\left(\frac{A_1}{A_2}\right)^2 v_1^2 \tag{14.32}$$

FIGURE 14.31 A Venturi tube measures gas-flow speeds.



The potential energy terms have disappeared because $y_1 = y_2$ for a horizontal tube. Equation 14.32 can now be solved for v_1, then v_2 is obtained from Equation 14.30. We'll skip a few algebraic steps and go right to the result:

$$v_1 = A_2 \sqrt{\frac{2\rho_{liq} g h}{\rho(A_1^{\,2} - A_2^{\,2})}}$$
$$v_2 = A_1 \sqrt{\frac{2\rho_{liq} g h}{\rho(A_1^{\,2} - A_2^{\,2})}} \qquad (14.33)$$

Equations 14.33 are reasonably accurate as long as the flow speeds are much less than the speed of sound, about 340 m/s. The Venturi tube is an example of the power of Bernoulli's equation.

As a final example, we can use Bernoulli's equation to understand, at least qualitatively, how airplane wings generate *lift*. FIGURE 14.32 shows the cross section of an airplane wing. This shape is called an *airfoil*.

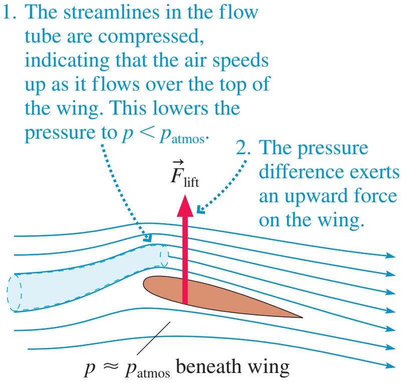
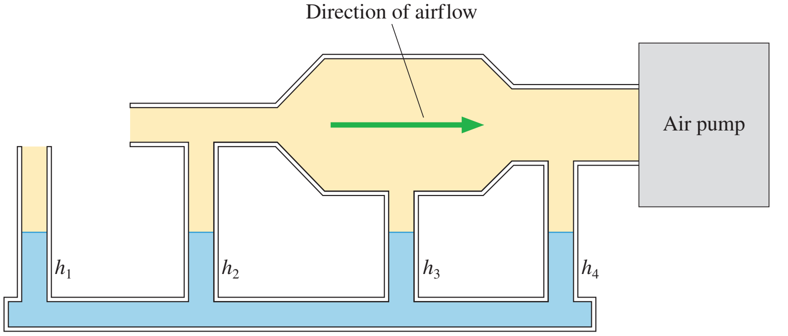


FIGURE 14.32 Airflow over a wing generates lift by creating unequal pressures above and below.

Although you usually think of an airplane moving through the air, in the airplane's reference frame it is the air that flows across a stationary wing. As it does, the streamlines must separate. The bottom of the wing does not significantly alter the streamlines going under the wing. But the streamlines going over the top of the wing get bunched together. As we've seen, with the equation of continuity, the flow speed has to increase when streamlines get closer together. Consequently, the air speed increases as it flows across the top of the wing.

If the air speed increases, then, from Bernoulli's equation, the air pressure must decrease. And if the air pressure above the wing is less than the air pressure below, the air will exert a net *upward* force on the wing. The upward force of the air due to the pressure difference across the wing is called **lift.** A full understanding of lift in aerodynamics involves other, more complicated factors, such as the creation of vortices on the trailing edge of the wing, but our introduction to fluid dynamics has given you enough tools to at least begin to understand how airplanes stay aloft.

STOP TO THINK 14.6 Rank in order, from highest to lowest, the liquid heights h_1 to h_4. The airflow is from left to right.



14.6 Motion of a Viscous Fluid

Bernoulli's equation, a statement about the energy of flowing fluids, is equivalent to our earlier discovery that mechanical energy is conserved for a system that has no dissipative forces. But nearly all real-world systems do have dissipative forces that generate thermal energy. Flowing fluids dissipate energy due to their *viscosity*.

Viscosity η, a fluid's resistance to flow, was introduced in « SECTION 6.5 when we looked at the drag force on an object moving through a stationary fluid. Now we want to analyze a moving fluid in a stationary tube. We'll restrict our analysis to tubes that have circular cross sections. TABLE 14.3 gives values of η for some common fluids. Note that viscosity decreases *very* rapidly with an increase in temperature.

TABLE 14.3 Viscosities of fluids

Fluid	η (Pa s)
Ethyl alcohol (20°C)	1.3×10^{-3}
Water (20°C)	1.0×10^{-3}
Water (40°C)	6.5×10^{-4}
Milk (20°C)	3.0×10^{-3}
Whole blood (37°C)	3.5×10^{-3}
Olive oil (20°C)	8.4×10^{-2}
Motor oil (20°C)	2.4×10^{-1}
Motor oil (100°C)	8.0×10^{-3}

Viscosity has a profound effect on how a fluid moves through a tube. **FIGURE 14.33a** shows that in an ideal fluid, which has no viscosity, all the fluid particles move with the same speed v, the speed that appears in Bernoulli's equation. For a viscous fluid, seen in **FIGURE 14.33b**, the fluid moves fastest in the center of the tube. The speed decreases away from the center until it reaches zero on the walls of the tube; that is, the fluid in contact with the walls of the tube does not move at all. Whether it is water moving through pipes or blood through arteries, the fact that the fluid at the outer edges "lingers" and barely moves allows deposits to build up on the inside walls of a tube.

We can't characterize the flow of a viscous fluid by a single speed v, but we can define an *average* flow speed v_{avg}. The volume flow rate $Q = \Delta V/\Delta t$ is still a well-defined quantity, so, corresponding to $Q = vA$ for an ideal fluid, we define the average flow speed of a viscous fluid through a tube with cross-section area A by

$$v_{avg} = \frac{Q}{A} \tag{14.34}$$

FIGURE 14.34 shows fluid flowing smoothly (i.e., laminar flow) with average speed v_{avg} through a horizontal tube that has constant radius r. There's no change of height ($y_2 = y_1$) and no change of speed along a streamline between points 1 and 2 ($v_2 = v_1$). If this is an ideal fluid, then we learn from Bernoulli's equation that $\Delta p = p_1 - p_2 = 0$. That is, there's no net force on this segment of fluid; it simply "coasts" through the tube at constant speed with no change in pressure. It's equivalent to a puck gliding at constant speed across a frictionless surface with no applied force.

If there's friction, however, something has to apply a steady force—equal and opposite the friction force—to keep the puck moving at a steady speed. Likewise, something has to apply a steady force—equal and opposite the viscous drag—to push a viscous fluid through a tube at a steady speed. That "something" is the pressure difference Δp between the ends of the tube. *A pressure difference is needed to keep a viscous fluid flowing,* whereas no pressure difference is needed for an ideal fluid. The net force on the segment of fluid in the tube of Figure 14.34 is $F_{net} = A\,\Delta p$.

We can use the energy principle to determine how large the pressure difference must be. Our starting point for finding Bernoulli's equation was $W_{ext} = \Delta K + \Delta U$, which is valid for a system that has no dissipative forces. Dissipative forces transform mechanical energy into thermal energy, which leads to $W_{ext} = \Delta K + \Delta U + \Delta E_{th}$. In « **SECTION 9.5**, we found that $\Delta E_{th} = f_k\,\Delta x$ when friction is the dissipative force. This is easily generalized: If F_{drag} is the "friction" due to a fluid's viscosity, then pushing the fluid a distance Δx against the viscous drag generates thermal energy $\Delta E_{th} = F_{drag}\,\Delta x$.

We will limit our analysis to the horizontal tube of Figure 14.34, in which case $\Delta U_G = 0$ and also $\Delta K = 0$ because v_{avg} doesn't change in a tube of constant diameter. We've already calculated the net work done by a pressure difference: Equation 14.23 found $W_{ext} = \Delta p V = \Delta p A \Delta x$ for the work done to push the fluid through Δx. Thus the energy principle for a viscous fluid flowing through this horizontal tube is

$$W_{ext} = \Delta p A \Delta x = \Delta E_{th} = F_{drag}\,\Delta x \tag{14.35}$$

The work done by the pressure difference does not increase the fluid's kinetic energy or potential energy, only the thermal energy of the fluid and the tube.

In Section 6.5 we found that the drag on a sphere of radius r moving through a fluid with viscosity η is $F_{drag} = 6\pi\eta v r$. We see that drag depends linearly on the viscosity, the speed, and the distance over which viscous forces act. Thus it would seem likely that the viscous drag on a segment of fluid flowing through a tube of length L is $F_{drag} = c\eta v_{avg} L$, where c is an unknown constant. Delving deeper into the fundamentals of fluid mechanics, it can be shown that $c = 8\pi$ and thus $F_{drag} = 8\pi\eta v_{avg} L$. Then, using this in Equation 14.35, we find that the pressure difference needed to push a viscous fluid through a tube with average speed v_{avg} is

$$\Delta p = \frac{8\eta v_{avg} L}{r^2} \tag{14.36}$$

where $A = \pi r^2$ is used as the cross-section area. A larger pressure difference is needed for a more viscous fluid, a longer tube, or a narrower tube.

FIGURE 14.33 Viscosity alters the speed profile of a fluid flowing through a tube.

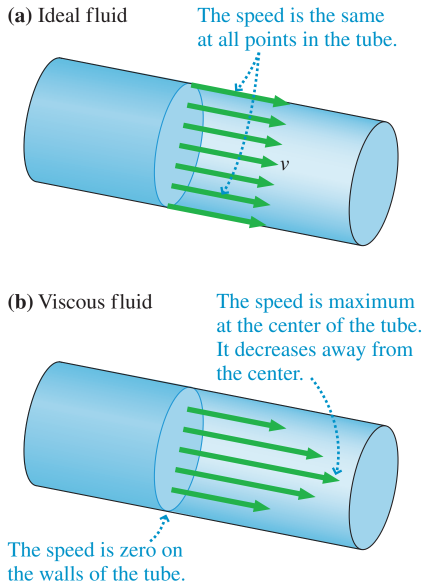
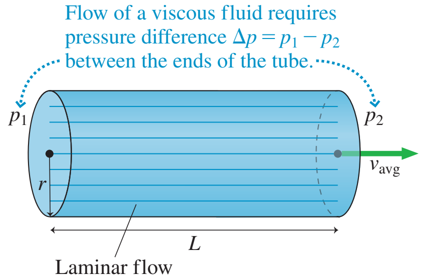


FIGURE 14.34 Laminar flow of a viscous fluid requires a pressure difference.



Solving for v_{avg}, we find that a pressure difference Δp across a tube of length L causes a fluid to flow with average speed

$$v_{avg} = \frac{r^2}{8\eta}\frac{\Delta p}{L} \tag{14.37}$$

Not surprisingly, a more viscous fluid flows more slowly.

The more practical quantity, because it's easily measured, is the volume flow rate Q. Because $Q = v_{avg}A = \pi r^2 v_{avg}$, the flow rate is

$$Q = \frac{\pi r^4}{8\eta}\frac{\Delta p}{L} \tag{14.38}$$

This is called **Poiseuille's equation** for viscous flow, named for a 19th-century French scientist who studied fluids and was especially interested in blood flow.

The quantity $\Delta p/L$ is called the **pressure gradient.** It is the pressure change per unit length or, equivalently, the slope of a pressure-versus-distance graph. A large pressure change over a small distance is a large pressure gradient. We can interpret Poiseuille's equation as saying that *a pressure gradient drives a fluid flow*. This is our first introduction to the idea that **gradients drive flows**, but it won't be our last.

One surprising consequence of Poiseuille's equation is the very strong dependence of the flow on the tube's radius: The flow rate is proportional to the *fourth* power of r. Fairly small changes in the radius have a large effect on the flow rate.

EXAMPLE 14.13 ■ Blood flow through a clogged artery

Atherosclerosis is a condition in which an artery is narrowed by the buildup of fatty plaques on its inner wall (as shown in the photo). The process restricts blood flow to organs, including the heart, and can lead to serious health issues. Suppose the diameter of an artery is reduced by 25%, a not untypical value. By what percent is the blood flow reduced if the pressure difference stays constant? By what factor would the pressure difference established by the heart have to increase to keep the blood flow rate constant?

MODEL Model the blood flow through the artery as viscous flow through a tube of constant diameter.

SOLVE Suppose the flow rate is Q_1 through an unclogged artery of radius r_1 and Q_2 through a clogged artery with a smaller radius $r_2 = 0.75r_1$. From Poiseuille's equation, Equation 14.38, we find

$$\frac{Q_2}{Q_1} = \frac{{r_2}^4}{{r_1}^4} = (0.75)^4 = 0.32$$

A 25% reduction in diameter leads to a huge 68% reduction in flow if the pressure difference is unchanged. Conversely, for a constant flow rate, the pressure difference Δp is inversely proportional to r^4. Thus

$$\frac{\Delta p_2}{\Delta p_1} = \frac{(1/r_2)^4}{(1/r_1)^4} = \left(\frac{1}{0.75}\right)^4 = 3.2$$

Blood pressure would have to increase by a factor of 3.2 to maintain a constant flow rate.

REVIEW The fourth-power dependence on r has profound consequences for the flow of a viscous fluid. Physiologically, the heart is not able to substantially increase the pressure, so atherosclerosis causes a significant reduction of blood flow. It is the leading cause of death in the United States.

EXAMPLE 14.14 ■ Measuring viscosity

A liquid with density 1200 kg/m^3 flows through the horizontal tube shown in FIGURE 14.35. The flow rate is measured to be 7.5 L/min. What is the liquid's viscosity?

MODEL Treat the flow as the flow of a viscous fluid.

VISUALIZE The 8.1 cm difference in liquid heights in the vertical tubes shows that there's a pressure difference and thus a pressure gradient between points $L = 75$ cm $= 0.75$ m apart. The tube's radius is $r = 0.50$ cm $= 5.0 \times 10^{-3}$ m.

FIGURE 14.35 Fluid flowing through a constant-diameter tube.