GLOBAL EDITION

Microeconomics

Theory and Applications with Calculus

FIFTH EDITION

Jeffrey M. Perloff

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Factor Price Changes. Once the beer manufacturer determines the lowest-cost combination of inputs to produce a given level of output, it uses that method as long as the input prices remain constant. How should the firm change its behavior if the cost of one of the factors changes?

Suppose that the wage falls from \$24 to \$8 but the rental rate of capital stays constant at \$8. Because of the wage decrease, the new isocost line in Figure 7.5 has a flatter slope, $-w/r = -8/8 = -1$, than the original isocost line, $-w/r = -24/8 = -3$. The change in the wage does not affect technological efficiency, so it does not affect the isoquant. The relatively steep original isocost line is tangent to the 100-unit isoquant at point x ($L = 50, K = 100$), while the new, flatter isocost line is tangent to the isoquant at v ($L = 77, K = 52$). Because labor is now relatively less expensive, the firm uses more labor and less capital. Moreover, the firm's cost of producing 100 units falls from \$2,000 to \$1,032 as a result of the decrease in the wage. This example illustrates that a change in the relative prices of inputs affects the combination of inputs that a firm selects and its cost of production.

Formally, we know from Equation 7.10 that the ratio of the factor prices equals the ratio of the marginal products: $w/r = MP_L/MP_K$. As we have already determined, this expression is $w/r = 1.5K/L$ for the beer manufacturer. Holding r fixed for a small change in w, the change in the factor ratio is $\text{d}(K/L)/\text{d}w = 1/(1.5r)$. For the beer manufacturer, where $r = 8$, $\text{d}(K/L)/\text{d}w = 1/12 \approx 0.083$. Because this derivative is positive, a small change in the wage leads to a higher capital-labor ratio because the firm substitutes some relatively less expensive capital for labor.

How Long-Run Cost Varies with Output

We now know how a firm determines the cost-minimizing combination of inputs for any given level of output. By repeating this analysis for different output levels, the firm determines how its cost varies with output.

Figure 7.5 Change in Factor Price

Originally the wage was \$24 and the rental rate of capital was \$8, so the lowest isocost line (\$2,000) was tangent to the $q = 100$ isoquant at x ($L = 50, K = 100$). When the wage fell to \$8, the isocost lines became flatter: Labor became relatively less expensive than capital. The slope of the isocost lines falls from $-w/r = -24/8 = -3$ to $-8/8 = -1$. The new lowest isocost line (\$1,032) is tangent at v ($L = 77, K = 52$). Thus, when the wage falls, the firm uses more labor and less capital to produce a given level of output, and the cost of production falls from \$2,000 to \$1,032.

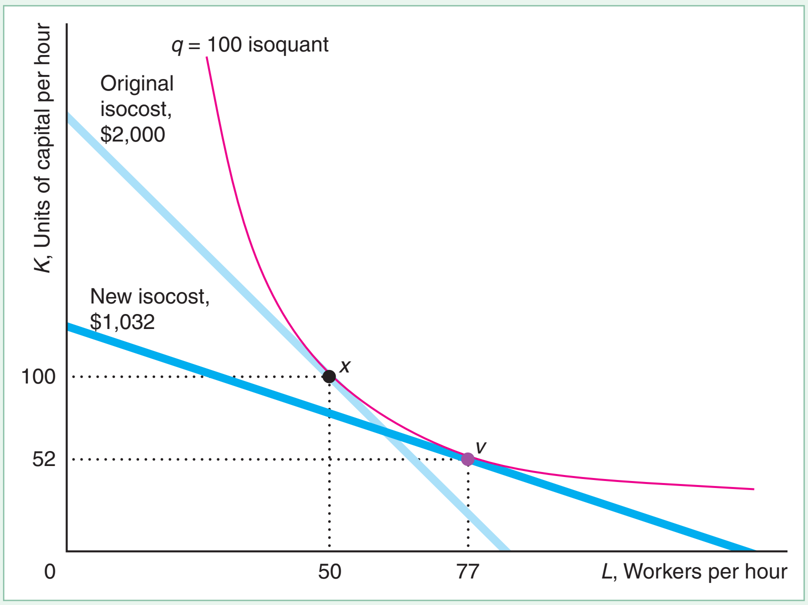
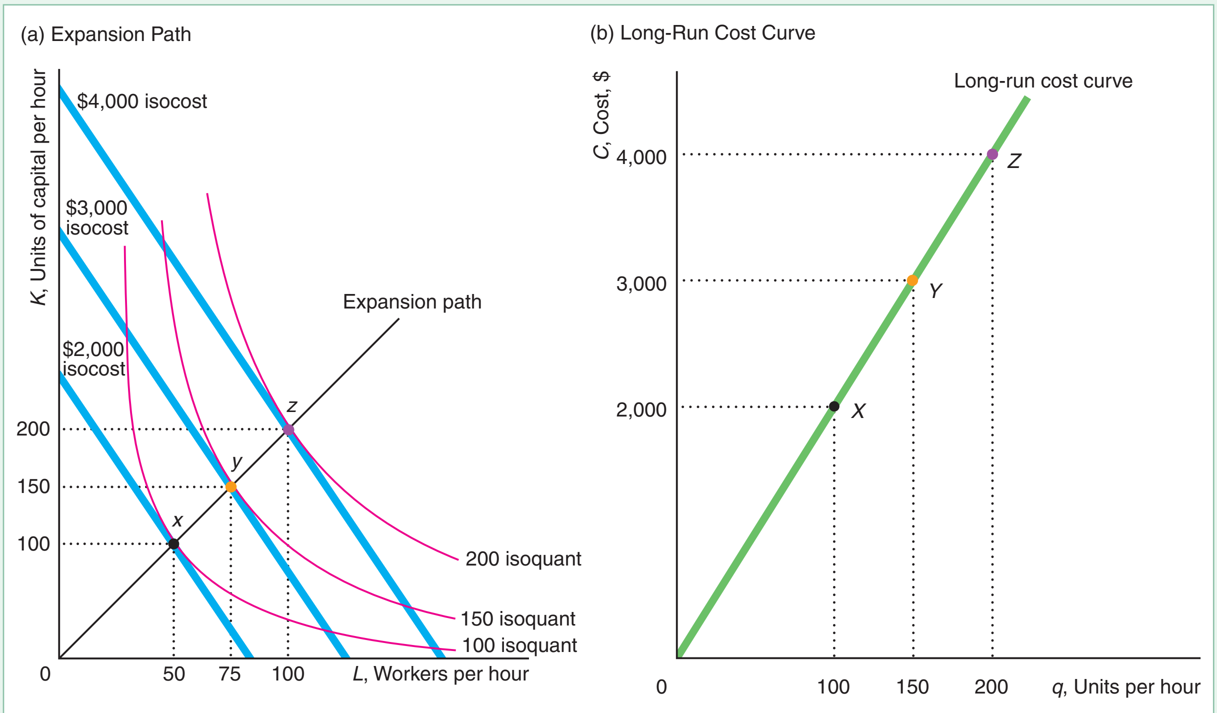


Figure 7.6 Expansion Path and Long-Run Cost Curve

(a) The curve through the tangency points between isocost lines and isoquants, such as x, y, and z, is called the expansion path. The points on the expansion path are the cost-minimizing combinations of labor and capital for each output level. (b) The beer manufacturer's expansion path shows the same relationship between long-run cost and output as the long-run cost curve.



Expansion Path. Panel a of Figure 7.6 shows the relationship between the lowest-cost factor combinations and various levels of output for the beer manufacturer when input prices are held constant at $w = \$24$ and $r = \$8$. The curve through the tangency points is the long-run **expansion path**: the cost-minimizing combination of labor and capital for each output level. The lowest-cost method of producing 100 units of output is to use the labor and capital combination x ($L = 50$ and $K = 100$), which lies on the $2,000 isocost line. Similarly, the lowest-cost way to produce 200 units is to use z, which lies on the $4,000 isocost line. The expansion path for the beer manufacturer is a straight line through the origin and x, y, and z, which has a slope of 2: At any given output level, the firm uses twice as much capital as labor. (In general, the expansion path need not be a straight line but can curve up or down as input use increases.)

SOLVED PROBLEM 7.5

MyLab Economics Solved Problem

What is the expansion path function for a constant-returns-to-scale Cobb-Douglas production function $q = AL^aK^{1-a}$? What is the path for the estimated beer manufacturer, which has a production function of $q = 1.52L^{0.6}K^{0.4}$?

Answer

Use the tangency condition between the isocost line and the isoquant that determines the cost-minimizing factor ratio to derive the expansion path. Because the marginal product of labor is $MP_L = aq/L$ and the marginal product of capital is $MP_K = (1 - a)q/K$, the tangency condition is

$$\frac{w}{r} = \frac{aq/L}{(1 - a)q/K} = \frac{a}{1 - a}\frac{K}{L}.$$

Using algebra to rearrange this expression, we obtain the expansion path formula:

$$K = \frac{(1 - a)}{a}\frac{w}{r}L. \qquad (7.25)$$

For the beer manufacturer in panel a of Figure 7.6, the expansion path, Equation 7.25, is $K = (0.4/0.6)(24/8)L = 2L$.

Long-Run Cost Function. The beer manufacturer's expansion path contains the same information as its long-run cost function, $C(q)$, which shows the relationship between the cost of production and output. As the expansion path plot in Figure 7.6 shows, to produce q units of output requires $K = q$ units of capital and $L = q/2$ units of labor. Thus, the long-run cost of producing q units of output is

$$C(q) = wL + rK = wq/2 + rq = (w/2 + r)q = (24/2 + 8)q = 20q.$$

That is, the long-run cost function corresponding to this expansion path is $C(q) = 20q$. This cost function is consistent with the expansion path in panel a: $C(100) = \$2,000$ at x on the expansion path, $C(150) = \$3,000$ at y, and $C(200) = \$4,000$ at z.

Panel b of Figure 7.6 plots this long-run cost curve. Points X, Y, and Z on the cost curve correspond to points x, y, and z on the expansion path. For example, the \$2,000 isocost line hits the $q = 100$ isoquant at x, which is the lowest-cost combination of labor and capital that can produce 100 units of output. Similarly, X on the long-run cost curve is at \$2,000 and 100 units of output. Consistent with the expansion path, the cost curve shows that as output doubles, cost doubles.

Solving for the cost function from the production function is not always easy. However, a cost function is relatively simple to derive from the production function if the production function is homogeneous of degree g so that $q = f(xL^*, xK^*) = x^g f(L^*, K^*)$, where x is a positive constant and L^* and K^* are particular values of labor and capital. That is, the production function has the same returns to scale for any given combination of inputs. Important examples of such production functions include the Cobb-Douglas ($q = AL^aK^b$, $g = a + b$), constant elasticity of substitution (CES), linear, and fixed-proportions production functions (see Chapter 6).

Because a firm's cost equation is $C = wL + rK$ (Equation 7.6), were we to double the inputs, we would double the cost. More generally, if we multiplied each output by x, the new cost would be $C = (wL^* + rK^*)x = \theta x$, where $\theta = wL^* + rK^*$. Solving the production function for x, we know that $x = q^{1/g}$. Substituting that expression in the cost equation, we find that the cost function for any homogeneous production function of degree g is $C = \theta q^{1/g}$. The constant in this cost function depends on factor prices and two constants, L^* and K^*. We would prefer to express the constant in terms of only the factor prices and parameters. We can do so by noting that the firm chooses the cost-minimizing combination of labor and capital, as summarized in the expansion path equation, as we illustrate in Solved Problem 7.6.

SOLVED PROBLEM 7.6

MyLab Economics Solved Problem

A firm has a Cobb-Douglas production function that is homogeneous of degree one: $q = AL^aK^{1-a}$. Derive the firm's long-run cost function as a function of only output and factor prices. What is the cost function that corresponds to the estimated beer manufacturer's production function $q = 1.52L^{0.6}K^{0.4}$?

Answer

1. *Combine the cost identity, Equation 7.6, with the expansion path, Equation 7.25, which shows how the cost-minimizing factor ratio varies with factor prices, to derive expressions for the inputs as a function of cost and factor prices.* From the expansion path, we know that $rK = wL(1 - a)/a$. Substituting for rK in the cost identity gives $C = wL + wL(1 - a)/a$. Simplifying shows that $L = aC/w$. Repeating this process to solve for K, we find that $K = (1 - a)C/r$.

2. *To derive the cost function, substitute these expressions of labor and capital into the production function.* By combining this information with the production function, we can obtain a relationship between cost and output. By substituting, we find that
$$q = A\left(\frac{aC}{w}\right)^a\left[\frac{(1-a)C}{r}\right]^{1-a}.$$
We can rewrite this equation as
$$C = \theta q, \tag{7.26}$$
where $\theta = w^a r^{1-a}/[Aa^a(1 - a)^{1-a}]$.

3. *To derive the long-run cost function for the beer firm, substitute the parameter values into* $C = \theta q$. For the beer firm, $C = [24^{0.6}8^{0.4}/(1.52 \times 0.6^{0.6}0.4^{0.4})]q \approx 20q$.

The Shape of Long-Run Cost Curves

The shapes of the average cost and marginal cost curves depend on the shape of the long-run cost curve. The relationships among total, marginal, and average costs are the same for both the long-run and short-run cost functions. For example, if the long-run average cost curve is U-shaped, the long-run marginal cost curve cuts it at its minimum.

The long-run average cost curve may be U-shaped, but the reason for this shape differs from those given for the short-run average cost curve. A key explanation for why the short-run average cost initially slopes downward is that the average fixed cost curve is downward sloping: Spreading the fixed cost over more units of output lowers the average fixed cost per unit. Because fixed costs are zero in the long run, fixed costs cannot explain the initial downward slope of the long-run average cost curve.

A major reason why the short-run average cost curve slopes upward at higher levels of output is diminishing marginal returns. In the long run, however, all factors can be varied, so diminishing marginal returns do not explain the upward slope of a long-run average cost curve.

As with the short-run curves, the shape of the long-run curves is determined by the production function relationship between output and inputs. In the long run, returns to scale play a major role in determining the shape of the average cost curve and the other cost curves. As we discussed in Chapter 6, increasing all inputs in proportion may cause output to increase more than in proportion (increasing returns to scale) at low levels of output, in proportion (constant returns to scale) at intermediate levels of output, and less than in proportion (decreasing returns to scale) at high levels of output. If a production function has this returns-to-scale pattern and the prices of inputs are constant, the long-run average cost curve must be U-shaped.

A cost function exhibits **economies of scale** if the average cost of production falls as output expands. Returns to scale is a sufficient condition for economies of scale. If a production function has increasing returns to scale, then the corresponding cost function has economies of scale: Doubling inputs more than doubles output, so average cost falls with higher output. However, if, for example, it is cost effective to switch to more capital-intensive production (such as using more robots and fewer workers) as the firm increases output, the cost function might exhibit economies of scale even if the production function does not have increasing returns to scale.

If an increase in output has no effect on average cost, then the production process has *no economies of scale*. Finally, a firm suffers from **diseconomies of scale** if average cost rises when output increases.

Average cost curves can have many different shapes. Perfectly competitive firms typically show U-shaped average cost curves. Average cost curves in noncompetitive markets may be U-shaped, L-shaped (average cost at first falls rapidly and then levels off as output increases), everywhere downward sloping, everywhere upward sloping, or take other shapes altogether. The shape of the average cost curve indicates whether the production process results in economies or diseconomies of scale. Some L-shaped average cost curves may be part of a U-shaped curve with long, flat bottoms, where we don't observe any firm producing enough to exhibit diseconomies of scale.

APPLICATION

3D Printing

Over the years, the typical factory has grown in size to take advantage of economies of scale to keep costs down. However, three-dimensional (3D) printing may reverse this trend by making it as inexpensive to manufacture one item as it is a thousand.

With 3D printing, an employee gives instructions—essentially a blueprint—to the machine, presses *Print*, and the machine builds the object from the ground up, either by depositing material from a nozzle or by selectively solidifying a thin layer of plastic or metal dust using drops of glue or a tightly focused beam.

Until recently, firms primarily used 3D printers to create prototypes in the aerospace, medical, and automotive industries. Then, they manufactured the final products using conventional techniques. However, costs have fallen to the point where manufacturing using 3D printers is often cost effective. According to Tucci Hot Rods (a car-customizing company) a hood vent for a Ford Fiesta ST modification costs $500 to build using machines and takes about three to four weeks to be delivered, compared to $15 to $17 in cost and 12 to 24 hours to 3D print.

Biomedical and aerospace companies are using 3D printing for just-in-time manufacturing. Printing can be used to fabricate small, highly customized batches of products as end-users need them. By 2018, Boeing and Airbus were each using thousands of different printed parts in their aircraft. The printers produce lighter parts, which lower the weight of planes (saving fuel) and can be quickly redesigned. Perhaps more striking, Airbus introduced the world's first entirely 3D-printed aircraft, a drone, in 2016. Some scientists and firms believe that 3D printing eventually will eliminate the need for many factories and may eliminate the manufacturing advantage of low-wage countries. Eventually, as the cost of printing drops, these machines may be used to produce small, highly customized batches as end-users need them.

Estimating Cost Curves Versus Introspection

Economists use statistical methods to estimate a cost function. However, we can sometimes infer the shape through casual observation and deductive reasoning.

For example, in the good old days, the Good Humor Company sent out herds of ice cream trucks to purvey its products. It seems likely that the company's production process had fixed proportions and constant returns to scale: If it wanted to sell more, Good Humor dispatched another truck and another driver. Drivers and trucks are almost certainly nonsubstitutable inputs (the isoquants are right angles). If the cost of a driver is w per day, the rental cost is r per day, and q is the quantity of ice cream sold per day, then the cost function is $C = (w + r)q$.

Such deductive reasoning can lead one astray, as I once discovered. A water-heater manufacturing firm provided me with many years of data on the inputs it used and the amount of output it produced. I also talked to the company's engineers about the production process and toured the plant (which resembled a scene from Dante's *Inferno,* with deafening noise levels and flames).

A water heater consists of an outside cylinder of metal, a liner, an electronic control unit, hundreds of tiny parts, and a couple of rods that slow corrosion. Workers cut out the metal for the cylinder, weld it together, and add the other parts. "Okay," I said to myself, "this production process must be one of fixed proportions because the firm needs one of each input to produce a water heater. How could you substitute a cylinder for an electronic control unit? Or substitute labor for metal?"

I then used statistical techniques to estimate the production and cost functions. Following the usual procedure, I did not assume that I knew the exact form of the functions. Rather, I allowed the data to "tell" me the type of production and cost functions. To my surprise, the estimates indicated that the production process was not one of fixed proportions. Rather, the firm could readily substitute between labor and metal.

"Surely I've made a mistake," I said to the plant manager after describing these results.

"No," he said, "that's correct. There's a great deal of substitutability between labor and metal."

"How can they be substitutes?"

"Easy," he said. "We can use a lot of labor and waste very little metal by cutting out exactly what we want and being very careful. Or we can use relatively little labor, cut quickly, and waste more metal. When the cost of labor is relatively high, we waste more metal. When the cost of metal is relatively high, we cut more carefully." This practice, as the manager explained, minimizes the firm's cost.

7.4 Lower Costs in the Long Run

In its long-term planning, a firm selects a plant size and makes other investments to minimize its long-run cost based on how many units it produces. Once it chooses its plant size and equipment, these inputs are fixed in the short run. Thus, the firm's long-run decisions determine its short-run cost. Because the firm cannot vary its capital in the short run but can in the long run, its short-run cost is at least as high as long-run cost and is higher if the "wrong" level of capital is used in the short run.

Long-Run Average Cost as the Envelope of Short-Run Average Cost Curves

As a result, the long-run average cost is always equal to or less than the short-run average cost. Figure 7.7 shows a firm with a U-shaped long-run average cost curve. Suppose initially that the firm has only three possible plant sizes. The firm's short-run