

GLOBAL  
EDITION



# STATISTICS FOR **BUSINESS AND ECONOMICS**

Tenth Global Edition

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# Statistics for Business and Economics

TENTH GLOBAL EDITION

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Pearson

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**Solution** You decide to use the monthly *return on stock price* over the 5-year period from May 2003 through April 2008. The stock-price data was obtained from Stock Investor Professional (Stock Investor Pro, 2007). The return on stock price was computed by dividing the change in month-end closing by the most recent month-end closing. Figure 5.27 contains the mean, variance, and covariance of the return on stock price for three firms—Infosys Technologies, Alcoa. Inc., Pearson PLC—for the 60 months from May 2003 through April 2008. Return on stock price is expressed as a proportion change for one month. This data are contained in the file **Return on Stock Price 60 month**. After a discussion with Susan, you decide to consider a portfolio whose dollar value includes 40% of Infosys, 30% of Alcoa, and 30% of Pearson. The mean value is computed using Equation 5.36 as follows:

$$\begin{aligned}\mu_W = E[W] &= (0.40)(0.0196) + (0.30)(0.00439) + (0.30)(0.00621) \\ &= 0.01101\end{aligned}$$

**Figure 5.27** Portfolio Statistics for Example 5.17 (Minitab Output)

**Descriptive Statistics: Infosys Tech, Alcoa Inc., Pearson PLC (ADR)**

Variable	N	Mean	StDev	Variance	Min	Median	Max
InfosysTech	60	0.0196	0.0926	0.0086	−0.2456	0.0254	0.1945
Alcoa Inc.	60	0.00439	0.07113	0.00506	−0.12813	0.01134	0.17137
Pearson PLC	60	0.00621	0.04655	0.00217	−0.09474	0.00391	0.10108

**Covariances: Infosys Tech, Alcoa Inc., Pearson PLC (ADR)**

	Infosys Tech	Alcoa Inc.	Pearson PLC
Infosys Tech	0.00857204		
Alcoa Inc.	0.00168845	0.00505950	
Pearson PLC	0.00086330	0.00150291	0.00216704

**Descriptive Statistics: Portfolio 1**

Variable	N	Mean	StDev	Variance	Min	Median	Max
Portfolio 1	60	0.01101	0.05390	0.00290	−0.13783	0.01950	0.15579

**Descriptive Statistics: S & P 500**

Variable	N	Mean	StDev	Variance	Min	Median	Max
SP 500	60	0.00655	0.02512	0.00063	−0.06515	0.01157	0.05210

Note that this portfolio mean is 1.101% per month, or 13.2% growth per year.

The variance is computed using Equation 5.37 as follows:

$$\begin{aligned}\sigma_W^2 &= (0.40^2)(0.0086) + (0.30^2)(0.00506) + (0.30^2)(0.00217) \\ &\quad + 2[(0.40)(0.30)(0.00168845) + (0.40)(0.30)(0.00086330) \\ &\quad + (0.30)(0.30)(0.00150291)] \\ &= 0.00290\end{aligned}$$

The standard deviation for the portfolio value is as follows:

$$\sigma_W = 0.05390$$

These computations can also be made by computing the value of the portfolio each month and then computing the mean and variance of the monthly portfolio values. The results are, of course, the same as shown for the variable, portfolio 1, in Figure 5.27. We have also included the mean and variance for the Standard and Poors (S & P) index for the same time period for perspective. Note that the mean growth ratio is higher for portfolio 1 compared to the S & P.

Assuming that stock price growth is normally distributed, we can also compute the probabilities that the total value of the portfolio is above or below particular values.

For example, the probability that the portfolio value is above 0.10 for one month can be determined by computing the standard normal  $Z$ :

$$Z_{0.10} = \frac{(0.10 - 0.01101)}{0.0539} = 1.651$$

The probability that the portfolio exceeds 0.10 is 0.049. We can also compute the probability that the portfolio value is less than 0.0 by first computing the standard normal  $Z$ :

$$Z_{0.0} = \frac{(0.0 - 0.01101)}{0.0539} = -0.204$$

The probability that the portfolio value is less than 0.0 is 0.081.

### Cautions Concerning Finance Models

The previous section introduced you to the basic understanding of the statistical modeling that is used in complex trading models. By using the means and variances computed for portfolios by these methods and the assumption of a normal distribution, analysts can determine the probability of various outcomes. This makes possible the determination of possible gains and losses adjusted by the probability of their occurrence. In addition, a number of very complex models have been developed—by extending the methods here—that use nonnormal probability distribution models and rigorous mathematical adjustments. These models have been developed by persons with strong mathematical skills, who may not have their basic education in business and economics. Computerized models are used successfully to assist very successful fund managers in their allocation of capital to the most appropriate uses.

There are, however, major examples where these models have failed with disastrous outcomes—such as the market collapse in September 2008. In some cases the models have been used to initiate trades with minimal human review. In the worst cases the models used are not well understood by the traders responsible for the funds, since the model developers may have departed from the company. The models work very well if the underlying probability distributions continue to be well approximated by the models. However, just because these approximations have occurred in the past, they are not guaranteed to occur in the future. Quoting again from Nassim Taleb, “. . . history teaches us that things that never happened before do happen.” Again, we encourage you to read his book, *Fooled by Randomness* (Taleb 2005). As noted in Chapter 3, Black Swans do occur with unknown probability. We cannot know the probability that the Chinese government will change the value of the yuan relative to the U.S. dollar, or that the U.S. dollar will lose its central role in world finance, or that some crazy people will destroy the World Trade Center. These and similar unknowns will have major and unpredictable effects on market outcomes.

Thus the wise analyst needs to constantly consider at least the following two ideas. First, under continuing stable conditions, the methods just developed will, if they are clearly understood, lead to better decisions. However, the unexpected can occur. Knowledge of probability can help one to understand that important fact. Second, one must also appreciate the fact that the unexpected could occur and lead to unexpected outcomes. If you have a broad understanding of world events you might be able to identify some possible unexpected events, but usually not their probabilities. We anticipate that patterns of events in the past will be followed by future similar patterns of events. But there is no guarantee. So, caution and continuous clear observation and thinking cannot be replaced by models developed from past events but must be used in combination with the models—which contain necessary, but not sufficient, analyses.

## Basic Exercises

- 5.61 A random variable  $X$  is normally distributed with a mean of 10 and a variance of 4, and a random variable  $Y$  is normally distributed with a mean of 15 and a variance of 25. The random variables have a correlation coefficient equal to 0.65. Find the mean and variance of the random variable:

$$W = 2X + 5Y$$

- 5.62 A random variable  $X$  is normally distributed with a mean of 28 and a variance of 1.44, and a random variable  $Y$  is normally distributed with a mean of 13 and a variance of 0.36. The random variables have a correlation coefficient equal to  $-0.2$ . Find the mean and variance of the random variable:

$$W = X + 4Y$$

- 5.63 A random variable  $X$  is normally distributed with a mean of 50 and a variance of 5.76, and a random variable  $Y$  is normally distributed with a mean of 20 and a variance of 9.61. The random variables have a correlation coefficient equal to 0.88. Find the mean and variance of the random variable:

$$W = \frac{1}{4}X + 3Y$$

- 5.64 A random variable  $X$  is normally distributed with a mean of 250 and a variance of 16.81, and a random variable  $Y$  is normally distributed with a mean of 360 and a variance of 13.69. The random variables have a correlation coefficient equal to 0.05. Find the mean and variance of the random variable:

$$W = -X + \sqrt{5}Y$$

- 5.65 A random variable  $X$  is normally distributed with a mean of 14.2 and a variance of 2.25, and a random variable  $Y$  is normally distributed with a mean of 13.5 and a variance of 0.81. The random variables have a correlation coefficient equal to 0.74. Find the mean and variance of the random variable:

$$W = X - 2Y$$

## Application Exercises

- 5.66 An investor plans to divide \$200,000 between two investments. The first yields a certain profit of 10%, whereas the second yields a profit with expected value 18% and standard deviation 6%. If the investor divides the money equally between these two investments, find the mean and standard deviation of the total profit.
- 5.67 A homeowner has installed a new energy-efficient furnace. It is estimated that over a year the new furnace will reduce energy costs by an amount that can be regarded as a random variable with a mean of \$200 and a standard deviation of \$60. Stating any assumptions you need to make, find the mean and standard deviation of the total energy cost reductions over a period of 5 years.
- 5.68 A consultant is beginning work on three projects. The expected profits from these projects are €65,000,

€83,000, and €57,000. The associated standard deviations are €12,500, €17,200, and €11,300. By assuming independence of the outcomes, find the mean and standard deviation of the consultant's total profit from these three projects.

- 5.69 A consultant has three sources of income—from teaching short courses, from selling computer software, and from advising on projects. His expected annual incomes from these sources are \$40,000, \$25,000, and \$18,000, and the respective standard deviations are \$2,000, \$5,000, and \$4,000. Assuming independence, find the mean and standard deviation of his total annual income.
- 5.70 Ten inspectors are employed by Norway's Kitron Group to check the quality of components produced on an assembly line. For each inspector the number of components that can be checked in a shift can be represented by a random variable with mean 150 and standard deviation 20. Let  $X$  represent the number of components checked by an inspector in a shift. Then the total number checked is  $10X$ , which has a mean of 1,500 and a standard deviation of 200. What is wrong with this argument? Assuming that inspectors' performances are independent of one another, find the mean and standard deviation of the total number of components checked in a shift.
- 5.71 It is estimated that in normal highway driving, the number of miles that can be covered by automobiles of a particular model on 1 gallon of gasoline can be represented by a random variable with mean 27 and standard deviation 2.5. Twenty-five of these cars, each with 1 gallon of gasoline, are driven independently under highway conditions. Find the mean and standard deviation of the average number of miles that will be achieved by these cars.
- 5.72 Shirley Johnson, portfolio manager, has asked you to analyze a newly acquired portfolio to determine its mean value and variability. The portfolio consists of 50 shares of Xylophone Music and 40 shares of Yankee Workshop. Analysis of past history indicates that the share price of Xylophone Music has a mean of 25 and a variance of 121. A similar analysis indicates that Yankee has a mean share price of 40 with a variance of 225. Your best evidence indicates that the share prices have a correlation of +0.5.
- Compute the mean and variance of the portfolio.
  - Suppose that the correlation between share prices was actually  $-0.5$ . Now what are the mean and variance of the portfolio?
- 5.73 Prairie Flower Cereal has annual sales revenue of \$400,000,000. George Severn, a 58-year-old senior vice president, is responsible for production and sales of Nougay 93 Fruity cereal. Daily production in cases is normally distributed, with a mean of 100 and a variance of 625. Daily sales in cases are also normally distributed, with a mean of 100 and a standard deviation

of 8. Sales and production have a correlation of 0.60. The selling price per case is \$10. The variable production cost per case is \$7. The fixed production costs per day are \$250.

- What is the probability that total revenue is greater than total costs on any day?
- Construct a 95% acceptance interval for total sales revenue minus total costs.

5.74 The nation of Olecarl, located in the South Pacific, has asked you to analyze international trade patterns. You first discover that each year it exports 10 units and imports 10 units of wonderful stuff. The price of exports is a random variable with a mean of 100 and a variance of 100. The price of imports is a random variable with a mean of 90 and a variance of 400. In addition, you discover that the prices of imports and exports have a correlation of  $\rho = -0.40$ . The prices of both exports and imports follow a normal probability density function. Define the balance of trade as the difference between the total revenue from exports and the total cost of imports.

- What are the mean and variance of the balance of trade?
- What is the probability that the balance of trade is negative?

5.75 You have been asked to determine the probability that the contribution margin for a particular product line

exceeds the fixed cost of \$1420. The total number of units sold is a normally distributed random variable with a mean of 400 and a variance of 900,  $X \sim N(400, 900)$ . The selling price per unit is \$9. The total number of units produced is a normally distributed random variable with a mean of 300 and a variance of 900  $Y \sim N(300, 900)$ . The variable production cost is \$6 per unit. Production and sales have a positive correlation of 0.50.

5.76 The nation of Waipo has recently created an economic development plan that includes expanded exports and imports. It has completed a series of extensive studies of the world economy and Waipo's economic capability, following Waipo's extensive 10-year educational-enhancement program. The resulting model indicates that in the next year exports will be normally distributed with a mean of 100 and a variance of 900 (in billions of Waipo yuan). In addition, imports are expected to be normally distributed with a mean of 105 and a variance of 625 in the same units. The correlation between exports and imports is expected to be +0.70. Define the trade balance as exports minus imports.

- Determine the mean and variance of the trade balance (exports minus imports) if the model parameters given above are true.
- What is the probability that the trade balance will be positive?

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## CHAPTER EXERCISES AND APPLICATIONS

5.77 A consultant knows that it will cost him \$10,000 to fulfill a particular contract. The contract is to be put out for bids, and he believes that the lowest bid, excluding his own, can be represented by a distribution that is uniform between \$8,000 and \$20,000. Therefore, if the random variable  $X$  denotes the lowest of all other bids

(in thousands of dollars), its probability density function is as follows:

$$f(x) = \begin{cases} 1/12 & \text{for } 8 < x < 20 \\ 0 & \text{for all other values of } x \end{cases}$$



- a. What is the probability that the lowest of the other bids will be less than the consultant's cost estimate of \$10,000?
  - b. If the consultant submits a bid of \$12,000, what is the probability that he will secure the contract?
  - c. The consultant decides to submit a bid of \$12,000. What is his expected profit from this strategy?
  - d. If the consultant wants to submit a bid so that his expected profit is as high as possible, discuss how he should go about making this choice.
- 5.78 The ages of a group of executives attending a convention are uniformly distributed between 35 and 65 years. If the random variable  $X$  denotes ages in years, the probability density function is as follows:

$$f(x) = \begin{cases} 1/30 & \text{for } 35 < x < 65 \\ 0 & \text{for all other values of } x \end{cases}$$

- a. Graph the probability density function for  $X$ .
  - b. Find and graph the cumulative distribution function for  $X$ .
  - c. Find the probability that the age of a randomly chosen executive in this group is between 40 and 50 years.
  - d. Find the mean age of the executives in the group.
- 5.79 The random variable  $X$  has probability density function as follows:

$$f(x) = \begin{cases} x & \text{for } 0 < x < 1 \\ 2 - x & \text{for } 1 < x < 2 \\ 0 & \text{for all other values of } x \end{cases}$$

- a. Graph the probability density function for  $X$ .
  - b. Show that the density has the properties of a proper probability density function.
  - c. Find the probability that  $X$  takes a value between 0.5 and 1.5.
- 5.80 An investment plan allows investors to deposit a minimum of £1,000 at the beginning of the term, which pays a fixed return rate of 5% per annum. After a year, investors have to deposit a minimum of £800 with an expected return rate of 3% per annum for the second year and a standard deviation of 2% per annum.
- a. Find the expected value of the total minimum amount earned after two years of investment.
  - b. Find the standard deviation of the total minimum amount earned after two years of investment.
- 5.81 A cobbler repairs shoes with normal damages for €2 each. The repaired shoes have a distribution with a mean of 60 and a standard deviation of 10.
- a. Find the mean daily total revenue earned from the shoes repaired.
  - b. Find the standard deviation of total revenues from the shoes repaired.
  - c. Daily costs (in euros) are given by

$$C = 80 + 0.50X$$

where  $X$  is the number of shoes repaired. Find the mean and standard deviation of daily profits from daily business.

- 5.82 A manufacturing team leader is evaluating the goods' production forecast times by comparing the actual times with the predicted times, where:

$$\text{actual time} = \text{predicted time} + \text{forecast error}$$

If the predicted time and forecast error are independent of each other, show that the variance of predicted time is less than the variance of actual time.

- 5.83 Let  $X_1$  and  $X_2$  be a pair of random variables. Show that the covariance between the random variables  $Y_1 = (2X_1 + X_2)$  and  $Y_2 = (2X_1 - X_2)$  is 3 if  $X_1$  and  $X_2$  have the same variance.

- 5.84 An ice-cream truck has an average daily profit of £250 with a standard deviation of £40.

- a. If a random business day is selected, what is the probability that the day's profit is more than £280?
- b. If a random business day is selected, what is the probability that the day's profit is between £240 and £265?
- c. What is the minimum amount of profit required for a day to be considered among the top 10% of most profitable business days?
- d. If a random sample of 200 business days is selected, what is the probability that at least 60 days have earned a daily profit of more than £280?
- e. If 2 business days are randomly selected, what is the probability that at least one day has a profit of more than £280?

- 5.85 Aircon, a South African company, services home air conditioners. It is known that times for service calls follow a normal distribution with mean of 50 minutes and a standard deviation of 12 minutes.

- a. What is the probability that a single service call takes more than 56 minutes?
- b. What is the probability that a single service call takes between 40 and 60 minutes?
- c. The probability is 0.05 that a single service call takes more than how many minutes?
- d. Find the shortest range of times that includes 40% of all service calls.
- e. A random sample of five service calls is taken. What is the probability that exactly three of them take more than 56 minutes?

- 5.86 It has been found that times taken by people to complete a particular tax form follow a normal distribution with a mean of 120 minutes and a standard deviation of 40 minutes.

- a. What is the probability that a randomly chosen person takes less than 100 minutes to complete this form?
- b. What is the probability that a randomly chosen person takes between 80 and 160 minutes to complete this form?
- c. One percent of all people take more than how many minutes to complete this form?

- d. Two people are chosen at random. What is the probability that at least one of them takes more than 90 minutes to complete this form?
- e. Five people are chosen at random. What is the probability that exactly three of them take longer than 90 minutes to complete this form?
- f. For the randomly chosen person, state in which of the following ranges (expressed in minutes) the time taken to complete the form is most likely to lie.  
100–120, 110–130, 120–140, 130–150
- g. For a randomly chosen person, state in which of the following ranges (expressed in minutes) the time to complete the form is least likely to lie.  
100–120, 110–130, 120–140, 130–150
- 5.87 Pizza EATaliano, a pizzeria in Budapest, delivers to residents of Vécsey street, Budapest. Delivery times follow a normal distribution with a mean of 30 minutes and a standard deviation of 6 minutes.
- a. What is the probability that a delivery will take between 25 and 35 minutes?
- b. The service does not charge for the pizza if delivery takes more than 45 minutes. What is the probability of getting a free pizza from a single order?
- c. During a national football tournament, a resident plans to order pizza six consecutive evenings. Assume that these delivery times are independent of each other. What is the probability that the resident will get at least one free pizza?
- d. Find the shortest range of times that includes 80% of all deliveries from this service.
- e. For a single delivery, state in which of the following ranges (expressed in minutes) the delivery time is most likely to lie.  
28–30, 29–31, 30–32, 31–33
- f. For a single delivery, state in which of the following ranges (expressed in minutes) the delivery time is least likely to lie.  
28–30, 29–31, 30–32, 31–33
- 5.88 Suppose data show that a country that has yearly expenses for research and development (R&D) follows a normal distribution with an average of \$10 billion. It also indicates that 15% of the countries spend more than \$20 billion on R&D per year. Find the percentage of the countries that spend more than \$25 billion on R&D in a year.
- 5.89 It is estimated that amounts of money spent on gasoline by customers at a gas station in Bristol, England, follow a normal distribution with a standard deviation £3.4. It is also found that 5% of all customers spent more than £30. What percentage of customers spent less than £25?
- 5.90 A market research organization has found that 45% of all supermarket shoppers refuse to cooperate when questioned by its pollsters. If 1,500 shoppers are approached, what is the probability that fewer than 700 will refuse to cooperate?
- 5.91 An organization that gives regular seminars on sales motivation methods determines that 55% of its clients have attended previous seminars. From a sample of 500 clients what is the probability that more than half attended previous seminars?
- 5.92 Helicopteros Sanitarios, an ambulance service in Fuengirola, Spain, receives an average of 20 calls per day during the time period 6 p.m. to 6 a.m. for assistance. For any given day what is the probability that fewer than 15 calls will be received during the 12-hour period? What is the probability that more than 25 calls during the 12-hour period will be received?
- 5.93 In a large department store a customer-complaints office handles an average of 12 complaints per hour about the quality of service. The distribution is Poisson.
- a. What is the probability that in any hour exactly six complaints will be received?
- b. What is the probability that more than 20 minutes will elapse between successive complaints?
- c. What is the probability that fewer than 5 minutes will elapse between successive complaints?
- d. The store manager observes the complaints office for a 30-minute period, during which no complaints are received. He concludes that a talk he gave to his staff on the theme “the customer is always right” has obviously had a beneficial effect. Suppose that, in fact, the talk had no effect. What is the probability of the manager observing the office for a period of 30 minutes or longer with no complaints?
- 5.94 A fish market in Phnom Penh, Cambodia, offers a large variety of fresh fish on its stands. You want to prepare the popular Khmer dish called amok and found that the average chunk of catfish, one of the main ingredients, on sale has a weight of 3.4 grams, with a standard deviation of 0.7 gram. Assuming the weights of catfish are normally distributed, what is the probability that a randomly selected Khmer dish with a piece of fish weighed more than 4 grams?
- 5.95 A Godiva Chocolate Shop in Germany offers different sizes and weights of boxes of truffles.
- a. Find the probability that a box of truffles weighs between 300 and 305 grams. The mean weight of a box is 300 grams and the standard deviation is 2.4 grams.
- b. After a more careful check, the standard deviation was found to be 2.5 grams. Find the new probability.
- 5.96 A management consultant found that the amount of time per day spent by executives performing tasks that could be done equally well by subordinates followed a normal distribution with a mean of 2.4 hours. It was also found that 10% of executives spent over 3.5 hours per day on tasks of this type. For a random sample of 400 executives, find the probability that more than 80 spend more than 3 hours per day on tasks of this type.