

# TABLE OF CONTENTS

CHAPTER	PAGE
I. INTRODUCTORY CONCEPTS	
§ 1. Binary operations	1
§ 2. Groups	3
§ 3. Subgroups	4
§ 4. Abelian groups	6
§ 5. Rings	7
§ 6. Rings with identity	8
§ 7. Powers and multiples	9
§ 8. Fields	10
§ 9. Subrings and subfields	10
§ 10. Transformations and mappings	12
§ 11. Group homomorphisms	13
§ 12. Ring homomorphisms	16
§ 13. Identification of rings	19
§ 14. Unique factorization domains	21
§ 15. Euclidean domains	22
§ 16. Polynomials in one indeterminate	24
§ 17. Polynomial rings	28
§ 18. Polynomials in several indeterminates	34
§ 19. Quotient fields and total quotient rings	41
§ 20. Quotient rings with respect to multiplicative systems	46
§ 21. Vector spaces	49
II. ELEMENTS OF FIELD THEORY	
§ 1. Field extensions	55
§ 2. Algebraic quantities	55
§ 3. Algebraic extensions	60
§ 4. The characteristic of a field	62
§ 5. Separable and inseparable algebraic extensions	65
§ 6. Splitting fields and normal extensions	72
§ 7. The fundamental theorem of Galois theory	80
§ 8. Galois fields	82
§ 9. The theorem of the primitive element	84

CHAPTER		PAGE
	§ 10. Field polynomials. Norms and traces	86
	§ 11. The discriminant	92
	§ 12. Transcendental extensions	95
	§ 13. Separably generated fields of algebraic functions	102
	§ 14. Algebraically closed fields	106
	§ 15. Linear disjointness and separability	109
	§ 16. Order of inseparability of a field of algebraic functions	113
	§ 17. Derivations	120
III. IDEALS AND MODULES		
	§ 1. Ideals and modules	132
	§ 2. Operations on submodules	136
	§ 3. Operator homomorphisms and difference modules	138
	§ 4. The isomorphism theorems	140
	§ 5. Ring homomorphisms and residue class rings	142
	§ 6. The order of a subset of a module	144
	§ 7. Operations on ideals	146
	§ 8. Prime and maximal ideals	149
	§ 9. Primary ideals	152
	§ 10. Finiteness conditions	155
	§ 11. Composition series	158
	§ 12. Direct sums	163
	§ 12 <sup>bis</sup> . Infinite direct sums	172
	§ 13. Comaximal ideals and direct sums of ideals	174
	§ 14. Tensor products of rings	179
	§ 15. Free joins of integral domains (or of fields)	187
IV. NOETHERIAN RINGS		
	§ 1. Definitions. The Hilbert basis theorem	199
	§ 2. Rings with descending chain condition	203
	§ 3. Primary rings	204
	§ 3 <sup>bis</sup> . Alternative method for studying the rings with d.c.c.	206
	§ 4. The Lasker-Noether decomposition theorem	208
	§ 5. Uniqueness theorems	210
	§ 6. Application to zero-divisors and nilpotent elements	213
	§ 7. Application to the intersection of the powers of an ideal	215
	§ 8. Extended and contracted ideals	218
	§ 9. Quotient rings	221
	§ 10. Relations between ideals in $R$ and ideals in $R_M$	223

# TABLE OF CONTENTS

xi

CHAPTER		PAGE
	§ 11. Examples and applications of quotient rings	227
	§ 12. Symbolic powers	232
	§ 13. Length of an ideal	233
	§ 14. Prime ideals in noetherian rings	237
	§ 15. Principal ideal rings	242
	§ 16. Irreducible ideals	247
	Appendix: Primary representation in noetherian modules	252
V. DEDEKIND DOMAINS. CLASSICAL IDEAL THEORY		
	§ 1. Integral elements	254
	§ 2. Integrally dependent rings	257
	§ 3. Integrally closed rings	260
	§ 4. Finiteness theorems	264
	§ 5. The conductor of an integral closure	269
	§ 6. Characterizations of Dedekind domains	270
	§ 7. Further properties of Dedekind domains	278
	§ 8. Extensions of Dedekind domains	281
	§ 9. Decomposition of prime ideals in extensions of Dedekind domains	284
	§ 10. Decomposition group, inertia group, and ramification groups	290
	§ 11. Different and discriminant	298
	§ 12. Application to quadratic fields and cyclotomic fields	312
	§ 13. A theorem of Kummer	318
	INDEX OF NOTATIONS	321
	INDEX OF DEFINITIONS	323