

CONTENTS

<i>Contents of Advanced Algebra</i>	x
<i>List of Figures</i>	xi
<i>Preface</i>	xiii
<i>Dependence Among Chapters</i>	xvii
<i>Standard Notation</i>	xviii
<i>Guide for the Reader</i>	xix
I. PRELIMINARIES ABOUT THE INTEGERS, POLYNOMIALS, AND MATRICES	1
1. Division and Euclidean Algorithms	1
2. Unique Factorization of Integers	4
3. Unique Factorization of Polynomials	9
4. Permutations and Their Signs	15
5. Row Reduction	19
6. Matrix Operations	24
7. Problems	30
II. VECTOR SPACES OVER \mathbb{Q}, \mathbb{R}, AND \mathbb{C}	33
1. Spanning, Linear Independence, and Bases	33
2. Vector Spaces Defined by Matrices	38
3. Linear Maps	42
4. Dual Spaces	50
5. Quotients of Vector Spaces	54
6. Direct Sums and Direct Products of Vector Spaces	58
7. Determinants	65
8. Eigenvectors and Characteristic Polynomials	73
9. Bases in the Infinite-Dimensional Case	77
10. Problems	82
III. INNER-PRODUCT SPACES	88
1. Inner Products and Orthonormal Sets	88
2. Adjoints	98
3. Spectral Theorem	104
4. Problems	111

IV. GROUPS AND GROUP ACTIONS	116
1. Groups and Subgroups	117
2. Quotient Spaces and Homomorphisms	128
3. Direct Products and Direct Sums	134
4. Rings and Fields	140
5. Polynomials and Vector Spaces	147
6. Group Actions and Examples	158
7. Semidirect Products	166
8. Simple Groups and Composition Series	170
9. Structure of Finitely Generated Abelian Groups	174
10. Sylow Theorems	183
11. Categories and Functors	188
12. Problems	198
V. THEORY OF A SINGLE LINEAR TRANSFORMATION	209
1. Introduction	209
2. Determinants over Commutative Rings with Identity	212
3. Characteristic and Minimal Polynomials	216
4. Projection Operators	224
5. Primary Decomposition	226
6. Jordan Canonical Form	229
7. Computations with Jordan Form	235
8. Problems	239
VI. MULTILINEAR ALGEBRA	245
1. Bilinear Forms and Matrices	246
2. Symmetric Bilinear Forms	250
3. Alternating Bilinear Forms	253
4. Hermitian Forms	255
5. Groups Leaving a Bilinear Form Invariant	257
6. Tensor Product of Two Vector Spaces	260
7. Tensor Algebra	274
8. Symmetric Algebra	280
9. Exterior Algebra	288
10. Problems	292
VII. ADVANCED GROUP THEORY	303
1. Free Groups	303
2. Subgroups of Free Groups	314
3. Free Products	319
4. Group Representations	326

VII. ADVANCED GROUP THEORY (Continued)

5. Burnside's Theorem	342
6. Extensions of Groups	344
7. Problems	357

VIII. COMMUTATIVE RINGS AND THEIR MODULES

1. Examples of Rings and Modules	367
2. Integral Domains and Fields of Fractions	378
3. Prime and Maximal Ideals	381
4. Unique Factorization	384
5. Gauss's Lemma	390
6. Finitely Generated Modules	396
7. Orientation for Algebraic Number Theory and Algebraic Geometry	408
8. Noetherian Rings and the Hilbert Basis Theorem	414
9. Integral Closure	417
10. Localization and Local Rings	425
11. Dedekind Domains	434
12. Problems	439

IX. FIELDS AND GALOIS THEORY

1. Algebraic Elements	449
2. Construction of Field Extensions	453
3. Finite Fields	457
4. Algebraic Closure	460
5. Geometric Constructions by Straightedge and Compass	464
6. Separable Extensions	469
7. Normal Extensions	476
8. Fundamental Theorem of Galois Theory	479
9. Application to Constructibility of Regular Polygons	483
10. Application to Proving the Fundamental Theorem of Algebra	486
11. Application to Unsolvability of Polynomial Equations with Nonsolvable Galois Group	488
12. Construction of Regular Polygons	493
13. Solution of Certain Polynomial Equations with Solvable Galois Group	501
14. Proof That π Is Transcendental	510
15. Norm and Trace	514
16. Splitting of Prime Ideals in Extensions	521
17. Two Tools for Computing Galois Groups	527
18. Problems	534

X. MODULES OVER NONCOMMUTATIVE RINGS	544
1. Simple and Semisimple Modules	544
2. Composition Series	551
3. Chain Conditions	556
4. Hom and End for Modules	558
5. Tensor Product for Modules	565
6. Exact Sequences	574
7. Problems	579
APPENDIX	583
A1. Sets and Functions	583
A2. Equivalence Relations	589
A3. Real Numbers	591
A4. Complex Numbers	594
A5. Partial Orderings and Zorn's Lemma	595
A6. Cardinality	599
<i>Hints for Solutions of Problems</i>	603
<i>Selected References</i>	697
<i>Index of Notation</i>	699
<i>Index</i>	703

CONTENTS OF *ADVANCED ALGEBRA*

I. Transition to Modern Number Theory
II. Wedderburn–Artin Ring Theory
III. Brauer Group
IV. Homological Algebra
V. Three Theorems in Algebraic Number Theory
VI. Reinterpretation with Adeles and Ideles
VII. Infinite Field Extensions
VIII. Background for Algebraic Geometry
IX. The Number Theory of Algebraic Curves
X. Methods of Algebraic Geometry