

Contents

1	Option Valuation and the Volatility Smile	1
1.1	Stochastic Processes for Stocks	1
1.1.1	Brownian Motion	1
1.1.2	Stock Price as Geometric Brownian Motion	3
1.1.3	Itô Process and Itô's Lemma	4
1.2	The Black-Scholes Model	5
1.2.1	Options and Dynamic Hedging	5
1.2.2	Risk-Neutral Valuation	7
1.2.3	Self-financing and No Arbitrage	8
1.2.4	Equivalent Martingale Measures	11
1.3	Volatility Quotations in Markets	15
1.3.1	Implied Volatilities	15
1.3.2	Market Quotations	16
1.3.3	Special Case: FX Market	17
2	Characteristic Functions in Option Pricing	21
2.1	Constructing Characteristic Functions (CFs)	22
2.1.1	A General Process for Stock Price	22
2.1.2	Valuation of European-style Options via CFs	23
2.1.3	Special Case: FX Options	27
2.2	Understanding Characteristic Functions	29
2.2.1	Properties of Characteristic Functions	29
2.2.2	Economic Interpretation of CFs	32
2.2.3	Examination of Existing Option Models	35
2.2.4	Relationship between CF to PDE	39
2.2.5	Advantages of CF and Modular Pricing	42
3	Stochastic Volatility Models	45
3.1	Introduction	45
3.2	The Heston Model	48
3.2.1	Model Setup and Properties	48

3.2.2	PDE Approach to Pricing Formula	50
3.2.3	Expectation Approach to Pricing Formula	52
3.2.4	Various Representations of CFs	54
3.3	The Schöbel-Zhu Model	55
3.3.1	Model Setup and Properties	55
3.3.2	Derivation of CFs	58
3.3.3	Numerical Examples	60
3.4	Double Square Root Model	63
3.4.1	Model Setup and Properties	63
3.4.2	Numerical Examples	69
3.5	Other Stochastic Volatility Models	70
3.6	Appendices	73
4	Numerical Issues of Stochastic Volatility Models	77
4.1	Alternative Pricing Formulas with CFs	78
4.1.1	The Formula à la Black-Scholes	78
4.1.2	The Carr and Madan Formula	78
4.1.3	The Attari Formula	80
4.2	Risk Sensitivities	80
4.2.1	Delta and Gamma	81
4.2.2	Various Vegas	82
4.2.3	Curvature and Slope	84
4.2.4	Volga and Vanna	84
4.3	Direct Integration (DI)	86
4.3.1	The Gaussian Integration	86
4.3.2	Multi-Domain Integration	87
4.3.3	Strike Vector Computation	88
4.4	Fast Fourier Transform (FFT)	89
4.4.1	Algorithms of FFT	89
4.4.2	Restrictions	91
4.5	Direct Integration vs. FFT	92
4.5.1	Computation Speed	93
4.5.2	Computation Accuracy	94
4.5.3	Matching Market Data	94
4.5.4	Calculation of Greeks	95
4.5.5	Implementation	95
4.6	Logarithm of Complex Number	99
4.6.1	Definition	99
4.6.2	Three Algorithms Dealing with Branch Cut	101
4.6.3	When Main Argument Is Appropriate	103
4.7	Calibration to Market Data	104
4.7.1	General Procedure	104
4.7.2	Fixing Velocity Parameter	105
4.7.3	Fixing Spot Volatility	106
4.8	Markovian Projection	108

5	Simulating Stochastic Volatility Models	113
5.1	Simulation Scheme	114
5.1.1	Discretization	114
5.1.2	Moment-Matching	115
5.2	Problems in the Heston Model	116
5.2.1	Negative Values in Paths	116
5.2.2	Log-normal Scheme	117
5.2.3	Transformed Volatility Scheme	118
5.2.4	QE Scheme	120
5.2.5	The Broadie-Kaya Scheme	122
5.2.6	Some Other Schemes	124
5.3	Simulation Examples	125
5.4	Maximum and Minimum	126
5.5	Multi-Asset Model	130
6	Stochastic Interest Models	135
6.1	Introduction	135
6.2	The Cox-Ingersoll-Ross Model	138
6.2.1	The Zero-Correlation Case	138
6.2.2	The Correlation Case	140
6.3	The Vasicek Model	142
6.4	The Longstaff Model	144
6.4.1	The Zero-Correlation Case	145
6.4.2	The Correlation Case	146
6.5	Correlations with Stock Returns: SI versus SV	148
7	Poisson Jumps	153
7.1	Introduction	153
7.2	Simple Jumps	158
7.3	Lognormal Jumps	160
7.4	Pareto Jumps	163
7.5	The Kou Model: An Equivalence to Pareto Jumps	165
7.6	Affine Jump-Diffusions	168
8	Lévy Jumps	173
8.1	Introduction	174
8.2	Stochastic Clock Models	177
8.2.1	Variance-Gamma Model	179
8.2.2	Normal Inverse Gaussian Model	181
8.3	Time-Changed Lévy Process	183
8.3.1	Uncorrelated Time-Change	184
8.3.2	Correlated Time-Change	188
8.4	The Barndorff-Nielsen and Shephard Model	193
8.5	Alpha Log-Stable Model	194
8.6	Empirical Performance of Various Lévy Processes	196

8.7	Monte-Carlo Simulation	198
8.7.1	Generating Random Variables	198
8.7.2	Simulation of Lévy Process	201
9	Integrating Various Stochastic Factors	203
9.1	Stochastic Factors as Modules	203
9.2	Integration Approaches	206
9.2.1	Modular Approach	206
9.2.2	Time-Change Approach	212
9.3	Pricing Kernels for Options and Bonds	217
9.4	Criteria for Model Choice	218
10	Exotic Options with Stochastic Volatilities	223
10.1	Forward-Starting Options	224
10.2	Barrier Options	226
10.2.1	Introduction	226
10.2.2	Two Special Cases	228
10.2.3	Numerical Examples	233
10.3	Lookback Options	235
10.3.1	Introduction	235
10.3.2	Pricing Formulas with Stochastic Factors	238
10.4	Asian Options	244
10.4.1	Introduction	244
10.4.2	The Black-Scholes World	245
10.4.3	Asian Options in a Stochastic World	249
10.4.4	Approximations for Arithmetic Average Asian Options	252
10.4.5	A Model for Asian Interest Rate Options	254
10.5	Correlation Options	257
10.5.1	Introduction	257
10.5.2	Exchange Options	260
10.5.3	Quotient Options	263
10.5.4	Product Options	264
10.6	Other Exotic Options	266
10.7	Appendices	267
11	Libor Market Model with Stochastic Volatilities	273
11.1	Introduction	273
11.2	Standard Libor Market Model	275
11.2.1	Model Setup	275
11.2.2	Term Structure and Smile of Volatility	280
11.3	Swap Market Model	282
11.3.1	Model Setup	282
11.3.2	Correlation Structure	286
11.3.3	Convexity Adjustments for CMS	291
11.4	Incorporating Stochastic Volatility	294

11.4.1 The Andersen and Brotherton-Ratcliffe Model	295
11.4.2 The Piterbarg Model	298
11.4.3 The Wu and Zhang Model	302
11.4.4 The Zhu Model	305
11.4.5 The Belomestny, Matthew and Schoenmakers Model	312
11.5 Conclusive Remarks	316
References	319
Index	327