

Contents

Volume I

Preface XXI

Part I

Boundary Value Problems for the Laplace Operator in Domains Perturbed Near Isolated Singularities

Chapter 1 Dirichlet and Neumann Problems for the Laplace Operator in Domains with Corners and Cone Vertices

1.1	Boundary Value Problems for the Laplace Operator in a Strip	3
1.1.1	The Dirichlet problem	3
1.1.2	The complex Fourier transform	5
1.1.3	Asymptotics of solution of the Dirichlet problem	6
1.1.4	The Neumann problem	7
1.1.5	Final remarks	8
1.2	Boundary Value Problems for the Laplace Operator in a Sector	8
1.2.1	Relationship between the boundary value problems in a sector and a strip	8
1.2.2	The Dirichlet problem	10
1.2.3	The Neumann problem	10
1.3	The Dirichlet Problem in a Bounded Domain with Corner	11
1.3.1	Solvability of the boundary value problem	11
1.3.2	Particular solutions of the homogeneous problem	13
1.3.3	Asymptotics of solution	15
1.3.4	A domain with a corner outlet to infinity	17
1.3.5	Asymptotics of the solutions for particular right-hand sides ...	18
1.3.6	The Dirichlet problem for the operator $\Delta - 1$	22
1.3.7	The Dirichlet problem in a domain with piecewise smooth boundary	26
1.4	The Neumann Problem in a Bounded Domain with a Corner	30
1.5	Boundary Value Problems for the Laplace Operator in a Punctured Domain and the Exterior of a Bounded Planar Domain	34
1.5.1	Dirichlet and Neumann problems in a punctured planar domain	34
1.5.2	Boundary value problems in the exterior of a bounded domain	36

1.6	Boundary Value Problems in Multi-Dimensional Domains	37
1.6.1	A domain with a conical point	37
1.6.2	A punctured domain	39
1.6.3	Boundary value problems in the exterior of a bounded domain	40
Chapter 2	Dirichlet and Neumann Problems in Domains with Singularly Perturbed Boundaries	
2.1	The Dirichlet Problem for the Laplace Operator in a Three-Dimensional Domain with Small Hole	44
2.1.1	Domains and boundary value problems	44
2.1.2	Asymptotics of the solution. The method of compound expansions	45
2.1.3	Asymptotics of the solution. The method of matched expansions	47
2.1.4	Comparison of asymptotic representations	51
2.2	The Dirichlet Problem for the Operator $\Delta - 1$ in a Three-Dimensional Domain with a Small Hole	52
2.3	Mixed Boundary Value Problems for the Laplace Operator in a Three-Dimensional Domain with a Small Hole	54
2.3.1	The boundary value problem with Dirichlet condition at the boundary of the hole	54
2.3.2	First version of the construction of asymptotics	55
2.3.3	Second version of the construction of asymptotics	57
2.3.4	The boundary value problem with the Neumann condition at the boundary of the gap	59
2.4	Boundary Value Problems for the Laplace Operator in a Planar Domain with a Small Hole	59
2.4.1	Dirichlet problem	60
2.4.2	Mixed boundary value problems	64
2.5	The Dirichlet Problem for the Operator $\Delta - 1$ in a Domain Perturbed Near a Vertex	67
2.5.1	Formulation of the problem	67
2.5.2	The first terms of the asymptotics	67
2.5.3	Admissible series	70
2.5.4	Redistribution of discrepancies	71
2.5.5	The set of exponents in the powers of ε , r , and ρ	72

Part II**General Elliptic Boundary Value Problems in Domains Perturbed
Near Isolated Singularities of the Boundary****Chapter 3 Elliptic Boundary Value Problems in Domains with Smooth
Boundaries, in a Cylinder, and in Domains with Cone Vertices**

3.1	Boundary Value Problems in Domains with Smooth Boundaries	79
3.1.1	The operator of an elliptic boundary value problem	79
3.1.2	Elliptic boundary value problems in Sobolev and Hölder spaces	80
3.1.3	The adjoint boundary value problem (the case of normal boundary conditions)	83
3.1.4	Adjoint operator in spaces of distributions	84
3.1.5	Elliptic boundary value problems depending on a complex parameter	85
3.1.6	Boundary value problems for elliptic systems	89
3.2	Boundary value problems in cylinders and cones	92
3.2.1	Solvability of boundary value problems in cylinders: the case of coefficients independent of t	92
3.2.2	Asymptotics at infinity of solutions to boundary value problems in cylinders with coefficients independent of t	95
3.2.3	Solvability of boundary value problems in a cone	97
3.2.4	Asymptotics of the solutions at infinity and near the vertex of a cone for boundary value problems with coefficients independent of r	99
3.2.5	Boundary value problems for elliptic systems in a cone	100
3.2.6	Asymptotics of the solution for the right-hand side given by an asymptotic expansion	104
3.3	Boundary Value Problems in Domains with Cone Vertices	106
3.3.1	Statement of the problem	106
3.3.2	Asymptotics of the solution near a cone vertex	107
3.3.3	Formulas for coefficients in the asymptotics of solution (under simplified assumptions)	109
3.3.4	Formula for coefficients in the asymptotics of solution (general case)	110
3.3.5	Index of the boundary value problem	113

**Chapter 4 Asymptotics of Solutions to General Elliptic Boundary Value
Problems in Domains Perturbed Near Cone Vertices**

4.1	Formulation of the Boundary Value Problems and some Preliminary Considerations	115
4.1.1	The domains	115
4.1.2	Admissible scalar differential operators	116
4.1.3	Limit operators	117
4.1.4	Matrices of differential operators	118
4.1.5	Boundary value problems	118
4.1.6	Function spaces with norms depending on the parameter ε	118

4.2	Transformation of the Perturbed Boundary Value Problem into a System of Equations and a Theorem about the Index	120
4.2.1	The limit operator	120
4.2.2	Reduction of the problem to a system	121
4.2.3	Reconstruction of the original problem from the system	124
4.2.4	Fredholm property for the operator of the boundary value problem in a domain with singularly perturbed boundary	127
4.2.5	On the index of the original problem	127
4.3	Asymptotic Expansions of Data in the Boundary Value Problem	130
4.3.1	Asymptotic expansion of the coefficients and the right-hand sides	131
4.3.2	Asymptotic formulas for solutions of the limit problems	132
4.3.3	Asymptotic expansions of operators of the boundary value problem	133
4.3.4	Preliminary description of algorithm for construction of the asymptotics of solutions	134
4.3.5	The set of exponents in asymptotics of solutions of the limit problems	137
4.3.6	Formal expansion for the operator in powers of small parameter	138
4.4	Construction and Justification of the Asymptotics of Solution of the Boundary Value Problem	140
4.4.1	The problem in matrix notation	140
4.4.2	Auxiliary operators and their properties	141
4.4.3	Formal asymptotics of the solution in the case of uniquely solvable limit problems	142
4.4.4	A particular basis in the cokernel of the operator M_0	144
4.4.5	Formal solution in the case of non-unique solvability of the limit problems	149
4.4.6	Asymptotics of the solution of the singularly perturbed problem	154
 Chapter 5 Variants and Corollaries of the Asymptotic Theory		
5.1	Estimates of Solutions of the Dirichlet Problem for the Helmholtz Operator in a Domain with Boundary Smoothened Near a Corner	157
5.2	Sobolev Boundary Value Problems	161
5.3	General Boundary Value Problem in a Domain with Small Holes	167
5.4	Problems with Non-Smooth and Parameter Dependent Data	173
5.4.1	The case of a non-smooth domain	173
5.4.2	The case of parameter dependent auxiliary problems	175
5.4.3	The case of a parameter independent domain	177
5.5	Non-Local Perturbation of a Domain with Cone Vertices	182
5.5.1	Perturbations of a domain with smooth boundary	182

5.5.2	Regular perturbation of a domain with a corner	184
5.5.3	A non-local singular perturbation of a planar domain with a corner	186
5.6	Asymptotics of Solutions to Boundary Value Problems in Long Tubular Domains	189
5.6.1	The problem	189
5.6.2	Limit problems	190
5.6.3	Solvability of the original problem	192
5.6.4	Expansion of the right-hand sides and the set of exponents in the asymptotics	193
5.6.5	Redistribution of defects	195
5.6.6	Coefficients in the asymptotic series	197
5.6.7	Estimate of the remainder term	198
5.6.8	Example	200
5.7	Asymptotics of Solutions of a Quasi-Linear Equation in a Domain with Singularly Perturbed Boundary	201
5.7.1	A three-dimensional domain with a small gap	202
5.7.2	A planar domain with a small gap	207
5.7.3	A domain smoothened near a corner point	213
5.8	Bending of an Almost Polygonal Plate with Freely Supported Boundary	217
5.8.1	Boundary value problems in domains with corners	219
5.8.2	A singularly perturbed domain and limit problems	220
5.8.3	The principal term in the asymptotics	221
5.8.4	The principal term in the asymptotics (continued)	223

Part III

Asymptotic Behaviour of Functionals on Solutions of Boundary Value Problems in Domains Perturbed Near Isolated Boundary Singularities

Chapter 6 Asymptotic Behaviour of Intensity Factors for Vertices of Corners and Cones Coming Close

6.1	Dirichlet's Problem for Laplace's Operator	228
6.1.1	Statement of the problem	228
6.1.2	Asymptotic behaviour of the coefficient C_ϵ^+	229
6.1.3	Justification of the asymptotic formula for the coefficient C_ϵ^+	230
6.1.4	The case $g \neq 0$	231
6.1.5	The two-dimensional case	231
6.2	Neumann's Problem for Laplace's Operator	232
6.2.1	Statement of the problem	232
6.2.2	Boundary value problems	232
6.2.3	The case of disconnected boundary	234
6.2.4	The case of connected boundary	235
6.3	Intensity Factors for Bending of a Thin Plate with a Crack	235
6.3.1	Statement of the problem	235
6.3.2	Clamped cracks (The asymptotic behaviour near crack tips)	236

6.3.3	Fixedly clamped cracks (Asymptotic behaviour of the intensity factors)	237
6.3.4	Freely supported cracks	238
6.3.5	Free cracks (The asymptotic behaviour of solution near crack vertices)	240
6.3.6	Free cracks (The asymptotic behaviour of intensity factors)	240
6.4	Antiplanar and Planar Deformations of Domains with Cracks	243
6.4.1	Torsion of a bar with a longitudinal crack	243
6.4.2	The two-dimensional problem of the elasticity theory in a domain with collinear close cracks	245
Chapter 7 Asymptotic Behaviour of Energy Integrals for Small Perturbations of the Boundary Near Corners and Isolated Points		
7.1	Asymptotic Behaviour of Solutions of the Perturbed Problem	251
7.1.1	The unperturbed boundary value problem	251
7.1.2	Perturbed problem	254
7.1.3	The second limit problem	254
7.1.4	Asymptotic behaviour of solutions of the perturbed problem	256
7.1.5	The case of right-hand sides localized near a point	259
7.2	Asymptotic Behaviour of a Bilinear Form	261
7.2.1	The asymptotic behaviour of a bilinear form (the general case)	261
7.2.2	Asymptotic behaviour of a bilinear form for right-hand sides localized near a point	264
7.2.3	Asymptotic behaviour of a quadratic form	266
7.3	Asymptotic Behaviour of a Quadratic Form for Problems in Regions with Small Holes	267
7.3.1	Statement of the problem	267
7.3.2	The case of uniquely solvable boundary problems	267
7.3.3	The case of the critical dimension	270
Chapter 8 Asymptotic Behaviour of Energy Integrals for Particular Problems of Mathematical Physics		
8.1	Dirichlet's Problem for Laplace's Operator	277
8.1.1	Perturbation of a domain near a corner or conic point	277
8.1.2	The case of right-hand sides depending on ξ	280
8.1.3	The case of right-hand sides depending on x and ξ	281
8.1.4	Dirichlet's problem for Laplace's operator in a domain with a small hole	282
8.1.5	Refinement of the asymptotic behaviour	284
8.1.6	Two-dimensional domains with a small hole	287
8.1.7	Dirichlet's problem for Laplace's operator in domains with several small holes	288

8.2	Neumann's Problem in Domains with one Small Hole	291
8.3	Dirichlet's Problem for the Biharmonic Equation in a Domain with Small Holes	293
8.4	Variation of Energy Depending on the Length of Crack	296
8.4.1	The antiplanar deformation	296
8.4.2	A problem in the two-dimensional elasticity	299
8.5	Remarks on the Behaviour of Solutions of Problems in the Two-dimensional Elasticity Near Corner Points	302
8.5.1	Statement of problems	302
8.5.2	The asymptotic behaviour of solutions of the antiplanar deformation problem	302
8.5.3	Asymptotic behaviour of solutions of the planar deformation problem	303
8.5.4	Boundary value problems in unbounded domains	306
8.6	Derivation of Asymptotic Formulas for Energy	308
8.6.1	Statement of problems	308
8.6.2	Antiplanar deformation	309
8.6.3	Planar deformation	310
8.6.4	Refinement of the asymptotic formula for energy	311
8.6.5	Defect in the material near vertex of the crack	313

Part IV

Asymptotic Behaviour of Eigenvalues of Boundary Value Problems in Domains with Small Holes

Chapter 9 Asymptotic Expansions of Eigenvalues of Classic Boundary Value Problems

9.1	Asymptotic Behaviour of the First Eigenvalue of a Mixed Boundary Value Problem	318
9.1.1	Statement of the problem	318
9.1.2	The three-dimensional case (formal asymptotic representation)	319
9.1.3	The planar case (formal asymptotic representation)	322
9.1.4	Justification of asymptotic expansions in the three-dimensional case	326
9.1.5	Justification of asymptotic expansions in the two-dimensional case	329
9.2	Asymptotic Expansions of Eigenvalues of Other Boundary Value Problems	331
9.2.1	Dirichlet's problem in a three-dimensional domain with a small hole	331
9.2.2	Mixed boundary value problem in domains with several small holes	334
9.2.3	Mixed boundary value problem with Neumann's condition on the boundary of small hole	337
9.2.4	Dirichlet's problem on a Riemannian manifold with a small hole	340

9.3	Asymptotic Representations of Eigenvalues of Problems of the Elasticity Theory for Bodies with Small Inclusions and Holes	342
9.3.1	Statement of the problem	342
9.3.2	Structure of the asymptotic representation	343
9.3.3	Particular solutions of the boundary layer problem	343
9.3.4	Perturbation of the eigenvalue Λ_0	347
9.3.5	Problem in the two-dimensional elasticity (one hole with a free surface)	349
Chapter 10	Homogeneous Solutions of Boundary Value Problems in the Exterior of a Thin Cone	
10.1	Formal Asymptotic Representation	355
10.1.1	Statement of the problem	355
10.1.2	The case $n - 1 > 2m$	356
10.1.3	The case $n - 1 = 2m$	360
10.2	Inversion of the Principal Part of an Operator Pencil on the Unit Sphere with a Small Hole. An Auxiliary Problem with Matrix Operator	363
10.2.1	“Nearly inverse” operator (the case $2m < n - 1$)	363
10.2.2	“Nearly inverse” operator (the case $2m = n - 1$)	367
10.2.3	Reduction to a problem with a matrix operator (the case $2m < n - 1$)	371
10.2.4	Reduction to a problem with a matrix operator (the case $2m = n - 1$)	373
10.3	Justification of the Asymptotic Behaviour of Eigenvalues (The Case $2m < n - 1$)	374
10.4	Justification of the Asymptotic Behaviour of Eigenvalues (The Case $2m = n - 1$)	379
10.5	Examples and Corollaries	388
10.5.1	A scalar operator	388
10.5.2	Lamé’s and Stokes’ systems	389
10.5.3	Continuity at the cone vertex of solution of Dirichlet’s problem	390
10.6	Examples of Discontinuous Solutions to Dirichlet’s Problem in Domains with a Conic Point	391
10.6.1	Equation of second order with discontinuous solutions	391
10.6.2	Dirichlet’s problem for an elliptic equation of the fourth order with real coefficients	393
10.7	Singularities of Solutions of Neumann’s Problem	395
10.7.1	Introduction	395
10.7.2	Formal asymptotic representation	396
10.8	Justification of the Asymptotic Formulas	400
10.8.1	Multiplicity of the spectrum near the point $\Lambda = 2$	400
10.8.2	Nearly inverse operator for Neumann’s problem in G_ε	401
10.8.3	Justification of asymptotic representation of eigenvalues	406

Comments on Parts I–IV

Comments on Part I	411
Chapter 1	411
Chapter 2	411
Comments on Part II	411
Chapter 3	411
Chapter 4	412
Chapter 5	412
Comments on Part III	412
Chapter 6	412
Chapter 7	408
Chapter 8	412
Comments on Part IV	412
Chapter 9	412
Chapter 10	412
List of Symbols	413
1. Basic Symbols	413
2. Symbols for function spaces and related concepts	414
3. Symbols for functions, distributions and related concepts	415
4. Other symbols	415
References	417
Index	433

Volume II

Preface	XXI
----------------------	-----

Part V
**Boundary Value Problems in Domains Perturbed Near
Multidimensional Singularities of the Boundary**
Chapter 11 Boundary Value Problems in Domains with Edges on the Boundary

11.1 The Dirichlet Problem for the Laplace Operator	3
11.1.1 Statement of the problem	3
11.1.2 Model problem in a wedge	4
11.1.3 Boundary value problem in Ω	8
11.2 The Neumann Problem for the Laplace Operator	9
11.2.1 Statement of the problem	9
11.2.2 Model problems	10
11.2.3 Solvability of the Neumann problem	14
11.2.4 The problem in a domain with a contour excluded	15
11.3 The Asymptotics Near an Edge of Solutions to Boundary Value Problems for the Laplace Operator	15
11.3.1 Estimates for the derivatives of solutions along an edge	15
11.3.2 The asymptotics of solutions to the Dirichlet problem	16
11.3.3 The asymptotics of solutions to the Neumann problem	18
11.3.4 The asymptotics of solutions to the boundary value problem in a domain with excluded contour	18
11.4 Sobolev Problems	19
11.4.1 Statement of the problem	19
11.4.2 Solvability of the problem	19
11.4.3 Asymptotics of a solution	20

**Chapter 12 Asymptotics of Solutions to Classical Boundary
Value Problems in a Domain with Thin Cavities**

12.1 The Asymptotics of a Solution of the Neumann Problem in the Exterior of a Thin Tube	23
12.1.1 Statement of the problem	23
12.1.2 Principal terms of asymptotics	24
12.1.3 Asymptotic series	30
12.2 The Asymptotics of Solutions to the Dirichlet Problem in the Exterior of a Thin Tube	35
12.2.1 Statement of the problem and the boundary layer	35
12.2.2 The integral equation on M	36
12.2.3 Justification of the asymptotics	39
12.2.4 Asymptotics of the capacity of a thin toroidal domain	41

12.3	Stress and Strain State of the Space with a Thin	
	Toroidal Inclusion	42
12.3.1	Statement of the problem	42
12.3.2	Preliminaries	42
12.3.3	Asymptotics of the solution	45
12.4	Asymptotics of Solutions to the Dirichlet Problem	
	in a Plane Domain with a Thin Cavity	48
12.4.1	Statement of the problem	48
12.4.2	A cavity whose shores encounter each other at zero angles	48
12.4.3	A thin cavity with smooth boundary	50
12.4.4	A remark on application of the results of Chapter 4	54
12.5	Asymptotics of Solutions to the Dirichlet Problem	
	in a Three-dimensional Domain with a Thin Cavity	55
12.5.1	Statement of the problem	56
12.5.2	Asymptotics of the solution in the exterior of $\bar{Q}_\varepsilon$	56
12.5.3	The two-dimensional boundary layer	57
12.5.4	Construction of the function γ	58
12.5.5	Boundary layer near the endpoints of segment M (the first term)	61
12.5.6	Boundary layer near the endpoints of the segment M (the second term)	63
12.5.7	Justification of the asymptotics	65
12.5.8	Asymptotics of capacity of the “ellipsoid” Q_ε	68
12.5.9	The case of a rotation ellipsoid	69
12.6	Smoothing the Boundary Near an Edge	70
12.6.1	The domain	71
12.6.2	The principal term of asymptotics	72
12.6.3	The complete asymptotic expansion	73
Chapter 13	Asymptotics of Solutions to the Dirichlet Problem for High	
	Order Equations in a Domain with a Thin Tube Excluded	
13.1	Statement of the Problem	76
13.2	The Case of Noncritical Dimension	77
13.3	The Case of Critical Dimension (Expansion in $(\log \varepsilon)^{-1}$)	81
13.4	The Case of Critical Dimension (Expansion in Powers of ε)	88
13.4.1	Structure of the asymptotic expansion	88
13.4.2	Structure of the formal series (1)	89
13.4.3	Asymptotic inversion of the operator $A_1 \log \varepsilon + A_2 + \hat{A}$	91
13.4.4	The power asymptotic series for the solution	95
13.4.5	More on the Dirichlet problem for the Laplace operator in the exterior of a tube	98

Part VI**Behaviour of Solutions of Boundary Value Problems in Thin Domains****Chapter 14 The Dirichlet Problem in Domains with Thin Ligaments**

14.1	The Principal Term in the Asymptotics of Solution	105
14.1.1	Statement of the problem (the case of two points on smooth surfaces approaching each other)	105
14.1.2	The limit boundary value problem	105
14.1.3	The asymptotics of solutions to the original problem	109
14.2	Complete Asymptotic Expansions of Solutions	111
14.2.1	The case of a quasicylindrical domain	111
14.2.2	Asymptotics of solutions to the Dirichlet problem	114
14.3	Asymptotics of Solutions for Nonsmooth Right-hand Side Terms	114
14.3.1	The second limit problem	114
14.3.2	Asymptotics of solutions	116
14.4	Ligaments of a Different Form	120
14.4.1	Two close conical points	120
14.4.2	Components of the boundary adhering at a large set	122
14.5	Asymptotics of the Condenser Capacity	124
14.5.1	Components of the boundary adhering at one point	124
14.5.2	Other condensers	126
14.5.3	The complete asymptotics of capacity	128

Chapter 15 Boundary Value Problems of Mathematical Physics in Thin Domains

15.1	Boundary Value Problems for the Laplace Operator in a Thin Rectangle	131
15.1.1	Statements of the problems	131
15.1.2	Asymptotics of solution to the Dirichlet problem	132
15.1.3	Asymptotics of solution to the mixed problem	133
15.2	The Principal Term in Asymptotics of the Solution to a Boundary Value Problem for a System of Second Order Equations in a Cylinder of Small Height	136
15.2.1	Statement of the problem	136
15.2.2	Auxiliary constructions	138
15.2.3	Asymptotics of the solution	140
15.2.4	Properties of the limit operator	142
15.2.5	Unique solvability of the limit problem	143
15.2.6	Justification of the asymptotic expansion of the solution	145
15.3	Applications of Theorem 15.2.9 to Particular Boundary Value Problems	149
15.4	Antiplanar Shear and Flow of an Ideal Fluid in a Thin Domain with a Longitudinal Cut	153
15.4.1	Statement of the problem	153
15.4.2	The two-dimensional case	154
15.4.3	Boundary layers	155

15.4.4	A supplementary limit problem and asymptotics of the intensity factor	157
15.4.5	The three-dimensional case	158
15.4.6	Examples	159
15.5	Intensity Factors for Close Parallel Cracks	161
15.5.1	Statement of the problem	161
15.5.2	Asymptotics of the solution inside and outside the strip between cracks	162
15.5.3	Boundary layers near tips of the cut	163
15.5.4	Estimate on the remainder in asymptotics	166
15.5.5	Asymptotics of the intensity factors	166
15.5.6	Shifted cracks	168
 Chapter 16 General Elliptic Problems in Thin Domains		
16.1	Limit Problems	171
16.1.1	Statement of the problem	171
16.1.2	Structure of differential operators	172
16.1.3	The ellipticity condition	173
16.1.4	The first limit problem	174
16.1.5	The second limit problem	175
16.1.6	The third limit problem	177
16.2	Asymptotics of Solutions	180
16.2.1	The Fredholm property of the original problem	180
16.2.2	The case when limit problems are uniquely solvable	182
16.2.3	Solutions to the third limit problem	185
16.2.4	Asymptotics in the case when $K + K^k > 0$	188
16.3	Examples	192
16.4	Bending of a Thin Plate	197
16.4.1	Statement of the problem	197
16.4.2	The first two limit problems	197
16.4.3	The complementary limit problem	198
16.4.4	The boundary layer	199
16.4.5	Boundary conditions in the third limit problem	206
16.4.6	Asymptotics of the solution	206

Part VII

Elliptic Boundary Value Problems with Oscillating Coefficients or Boundary of Domain

Chapter 17 Elliptic Boundary Value Problems with Rapidly Oscillating Coefficients

17.1	Homogenization of Differential Equation	211
17.1.1	Statement of the problem	211
17.1.2	The limit problem in the cell	212
17.1.3	The homogenized equation	213
17.1.4	Asymptotic series	215

17.2	Boundary Layer for the Dirichlet Problem	216
17.2.1	The boundary value problem for the boundary layer	216
17.2.2	Conditions on the boundary layer	218
17.3	Boundary Layer for the Neumann Problem	222
17.4	Justification of Asymptotic Expansions	226
17.5	Elliptic Boundary Value Problems with Periodic Coefficients	
	in a Cylinder	229
17.5.1	Model problem in a cylinder	229
17.5.2	Problem with complex parameter	230
17.5.3	Analog of the Fourier transform	231
17.5.4	Unique solvability of the model problem	233
17.5.5	Asymptotics of solutions	234
Chapter 18	Paradoxes of Limit Passage in Solutions of Boundary Value Problems When Smooth Domains Are Approximated by Polygons	
18.1	Approximation to a Freely Supported Convex Plate	238
18.1.1	Statement of the problem and description of the results	238
18.1.2	Formal asymptotics	239
18.1.3	Justification of the asymptotics	243
18.1.4	Concentrated moments at the vertices of the polygon	247
18.2	Approximation of a Hole in a Freely Supported Plate	249
18.2.1	Statement of the problem	249
18.2.2	Asymptotics of the solution for the plate with a polygonal hole	250
18.2.3	Point moments at the vertices of the polygon	252
18.3	Passage to Conditions of Rigid Support	254
Chapter 19	Homogenization of a Differential Operator on a Fine Periodic Net of Curves	
19.1	Statement of the Problem on a Net	259
19.1.1	The net S	259
19.1.2	The net S_ϵ	260
19.2	The Principal Term of Asymptotics	261
19.2.1	The formal asymptotics	261
19.2.2	Justification of the asymptotics	262
19.2.3	Asymptotics of solutions to nonstationary problems	266
19.3	Computation of Coefficients of the Homogenized Operator and Their Properties	267
19.3.1	Definition of coefficients of the homogenized operator	267
19.3.2	Ellipticity of the homogenized operator	269
19.3.3	Examples of homogenized operators	270
19.4	The Complete Asymptotic Expansion	273
19.4.1	Asymptotic solution outside a neighborhood of the boundary	273
19.4.2	Construction of the boundary layer	274
19.4.3	Definition of constants in the boundary layer	276

19.5 Asymptotics of the Solution to a Boundary Value Problem	
on a Net Located in a Cylinder	278
19.5.1 The boundary value problem in a cylinder	278
19.5.2 The boundary value problem on a cell	279
19.5.3 Asymptotic expansion of solutions to the problem	
in a cylinder	280
Chapter 20 Homogenization of Equations on a Fine Periodic Grid	
20.1 Homogenization of Difference Equations	283
20.1.1 A grid in $\mathbb{R}^n$ and the interaction set of its points	283
20.1.2 Statement of the problem	284
20.1.3 Solvability of the boundary value problem	285
20.1.4 The leading terms in asymptotics	286
20.1.5 Asymptotics of the solution to the	
nonstationary problem	288
20.2 Calculation of Coefficients of the Homogenized Operator	
and Their Properties	288
20.3 Crystalline Grid	291
20.3.1 Equations of the elasticity theory	291
20.3.2 Examples of homogenized operators	293
Comments on Parts V–VII	
Comments on Part V	297
Chapter 11	297
Chapter 12	297
Chapter 13	297
Comments on Part VI	298
Chapter 14	298
Chapter 15	298
Chapter 16	298
Comments on Part VII	298
Chapter 17	298
Chapter 18	299
Chapter 19	299
Chapter 20	299
List of Symbols	301
References	305
Index	321