

Contents

Preface	ii
Chapter I Background Ideas	1
Chapter II Elements of the Theory of Stochastic Processes	5
1. Brownian motion	5
2. Langevin equation and Fokker-Planck equation	7
3. Eigenvalue problem of the Fokker-Planck operator	12
4. Path-integral representation and randomization condition	15
5. Operator formalism	20
6. Perturbation theory	25
7. Generating functional and Green's function	27
Chapter III General Prescription of Stochastic Quantization	31
1. Basic ideas of SQM	31
2. Simple examples	35
2.1 Harmonic oscillator	35
2.2 Free neutral scalar field	37
2.3 Anharmonic oscillator and interacting field	39
3. Fermion field	41
4. Abelian gauge field	45
5. Finite temperature problem	47
6. Five-dimensional "stochastic" field theory for SQM	50
6.1 "Stochastic-canonical" field theory — "classical" formalism	51
6.2 "Stochastic-canonical" field theory — "operator" formalism	53
7. Generalized path-integral formulation	57
Chapter IV Perturbative Approach to Scalar Field Theory	62
1. Stochastic diagrams from Langevin equation	62
2. Stochastic diagrams from operator formalism	68
3. Reduction supersymmetry	74
Chapter V Perturbative Approach to Gauge Fields	78
1. Stochastic quantization without gauge fixing	78
1.1 Vacuum polarization tensor of QED	78

1.2 Gluon self-energy in non-Abelian gauge theory	81
2. Stochastic quantization with gauge fixing	87
2.1 Stochastic gauge fixing	87
2.2 Perturbation theory of non-Abelian gauge field with stochastic gauge fixing	89
2.3 Discussion on the Gribov problem	91
Chapter VI Stochastic Quantization of Constrained Systems	95
1. Stochastic quantization of constrained systems	95
2. Constrained Hamiltonian systems	100
2.1 Stochastic quantization in phase space	100
2.2 Systems with first class constraints	102
3. Stochastic quantization of compact gauge field	106
Chapter VII Superfield Formulation	108
1. Superfield formulation of stochastic quantization	108
2. Supersymmetry and Ward-Takahashi identities	110
3. Dimensional reduction	111
4. Connection with operator formalism	113
Chapter VIII Renormalization Scheme in Stochastic Quantization	117
1. General discussion	117
2. Power counting approach to renormalization	118
3. Superspace approach to renormalization	127
3.1 Superspace formulation of stochastic quantization	127
3.2 Renormalizability of the stochastic dynamics	129
3.3 Renormalization scheme and Ward identities	
— Scalar theory in 4-dimension	131
3.4 Problem of the boundary condition — twisted boundary condition	133
3.4.1 Superspace Feynman rules and boundary conditions	134
3.4.2 Determinant matching and boundary conditions	136
3.5 Higher order calculations	138
3.5.1 First order results	139
3.5.2 Second order contributions	140
4. Gauge theory	144
4.1 Generating functional and stochastic Ward identity	144
4.2 Gauge Ward identity and restricted gauge invariance	146
4.3 The background field method	147

4.3.1 The background gauge invariant stochastic generating functional	148
Chapter IX New Regularizations in Stochastic Quantization	154
1. General approach to regularization and fictitious-time-smearing regularization	154
2. Fictitious-time-smearing regularization II	158
3. Continuum regularization	160
Chapter X Generalized Langevin Equation and Anomaly	164
1. Generalized Langevin equation	164
1.1 Basic ideas of generalized Langevin equation	164
1.2 SU(N) lattice gauge theory	166
1.3 Fermion field theory	167
2. Anomaly	169
2.1 Chiral anomaly	169
2.2 Conformal anomaly	173
Chapter XI Application to Numerical Simulations	176
1. Basic procedure of Langevin simulation	176
2. Langevin source method	177
3. Nonlinear σ -model	178
4. Lattice QCD	179
5. Micro-canonical method	181
Chapter XII Minkowski Stochastic Quantization and Complex Langevin Equation	183
1. Langevin equation with a complex drift	183
2. Minkowski stochastic quantization	186
2.1 Naive Minkowski stochastic quantization	186
2.2 Use ofkerneled Langevin equations	191
3. Numerical application of the complex Langevin equation	194
3.1 Positivity of the Fokker-Planck operator	194
3.2 Blow-up solution	195
3.3 A kernel and unphysical solutions	197
3.4 Violation of ergodicity	200
Appendix A Differential and Integral Calculus of Grassmann Variables	204
1. Differentiation	204
2. Integration	205

Appendix B Stochastic Differential Calculus — Ito and Stratonovich Calculus	206
1. Wiener process and stochastic convergence	206
2. Ito calculus	207
3. Stratonovich calculus	209
References	211