

Contents

Introduction	vii
I Preliminaries	
A Lie Algebras	
§ 1 Basic facts	1
§ 2 Ideals, solvability, nilpotence, radical and semisimplicity	4
§ 3 S^p classes of solvable Lie algebras	8
§ 4 Radical splitting theorems	10
§ 5 Cartan subalgebras	11
§ 6 The Killing form and compact Lie algebras	15
§ 7 Lie $*$ -algebras	16
B Complexes	
§ 8 Generalities	23
§ 9 Banach space complexes	31
§ 10 Koszul complexes	33
§ 11 Operations with the Koszul complexes: duality and tensor products	52
C Spectral Theory in Complex Banach Space	
§ 12 General spectral theory and decomposable operators	63
§ 13 The transformation of the spectral maximal subspaces by bounded operators	68
§ 14 Special classes of decomposable operators	71
Notes	77
II The Commutators and Nilpotence Criteria	
§ 15 An asymptotic formula for the commutators	81
§ 16 Nilpotence criteria in an associative algebra	89
§ 17 Quasinilpotence and nilpotence criteria in complex Banach algebras	91
§ 18 Nilpotent elements in LM-decomposable Lie subalgebras of an associative algebra	95

§ 19 Nilpotent elements in LM-decomposable Lie algebras of bounded linear operators	101
Notes	101
III Infinite-dimensional Variants of the Lie and Engel Theorems	
§ 20 Weights for operator algebras	103
§ 21 Invariant subspaces for LM-decomposable Lie algebras of bounded operators	114
§ 22 The irreducible representations of an LM-decomposable Lie algebra. Infinite-dimensional variant of Lie's Theorem on a complex Banach space	121
§ 23 The associative envelope of a Lie algebra of quasinilpotent operators	126
§ 24 Commutativity modulo the Jacobson radical	129
Notes	131
IV Spectral Theory for Solvable Lie Algebras of Operators	
§ 25 Spectral theory for representations of Lie algebras	133
§ 26 Spectral theory for systems of operators generating nilpotent Lie algebras	150
§ 27 The Cartan-Taylor spectrum of a locally solvable Lie algebra of operators	163
§ 28 Lie ideals of generalized spectral operators	173
Notes	180
V Semisimple Lie Algebras of Operators	
§ 29 Lie subalgebra with involution consisting of bounded operators on a complex Banach space. Normal elements given by a space of self-adjoint operators	181
§ 30 Individual spectral properties in ideally finite semisimple Lie algebras of operators	188
§ 31 Semisimple Lie algebras of compact quasinilpotent operators	198
Notes	202
Bibliography	203
Index	215
List of Symbols	219