# Contents


**Preface** v

**Introduction** 1

**Chapter I General properties of nonlinear operators of Fejér type** 3

1 The main classes of nonlinear mappings . . . . . 3
- 1.1 (Quasi-)nonexpansive and pseudo-contractive operators . . . . . 3
- 1.2 Structure of the set of fixed points . . . . . 7
- 1.3 Existence of fixed points . . . . . 8

2 Strong convergence of iterations for quasi-nonexpansive operators . . . . . 9
- 2.1 The case of demi-compact operators. . . . . 10
- 2.2 The case of a linear operator . . . . . 11
- 2.3 Iterative processes for linear equations of the first kind . . . . . 12

3 Properties of iterations for the pseudo-contractive mappings 16
- 3.1 The main theorem on convergence of iterations. . . 16
- 3.2 Superpositions of pseudo-contractive operators. . . 19
- 3.3 Examples of the pseudo-contractive operators . . . 20

4 Iterations with the correcting multipliers . . . . . 25
- 4.1 Stability of fixed points on parameter . . . . . 25
- 4.2 One-step iterative process . . . . . 27
- 4.3 Asymptotic rule for stopping the iterations . . . . . 29

5 Fejér mappings and sequences. . . . . 31
- 5.1 Definitions and general properties . . . . . 31
- 5.2 Examples of Fejér mappings. . . . . 33
- 5.3 Weak-Fejér mappings and sequences . . . . . 35

6 Theorems on convergence of Fejér processes . . . . . 36
- 6.1 The case of the single-valued operator. . . . . 36



Bibliografische Informationen
http://d-nb.info/994977689

digitalisiert durch DEUTSCHE NATIONAL BIBLIOTHEK

6.2 The case of the multi-valued Fejér mappings . . . . 37

7 $M$-separating pairs and $M$-Fejér mappings . . . . . 39

**Chapter II Applications of iterative processes to nonlinear equations 43**

1 Gradient methods . . . . . 43

2 The Newton–Kantorovich method . . . . . 46

3 Fejér processes for mixed problems . . . . . 47
3.1 Systems of nonlinear equations and convex inequalities . . . . . 47
3.2 Linear case . . . . . 49

4 Nonlinear processes for linear operator equations . . . . . 50
4.1 Iterative $\alpha$-processes and extremal principles . . . . 50
4.2 Inequality for moments and pseudo-contractivity of the step operator . . . . . 52
4.3 Convergence of the $\alpha$-processes . . . . . 53

5 Linearized versions of the gradient methods . . . . . 55
5.1 The method of steepest descent . . . . . 55
5.2 Linearized analogue of the minimal error method . 57
5.3 Conclusions and applications . . . . . 58

6 The Levenberg–Marquardt method . . . . . 60
6.1 The idea of the method . . . . . 60
6.2 Weak convergence of the method . . . . . 60
6.3 Strong convergence of the modified method. Asymptotic rule for stopping the process . . . . . 63
6.4 Stopping the iterations by the residual . . . . . 65

7 Ill-posed problems with *a priori* information . . . . . 65
7.1 Formulation of the problem and convergence theorems . . . . . 65
7.2 Properties of iterations under noisy data . . . . . 69
7.3 Taking into account the *a priori* information in applied problems . . . . . 71

**Chapter III Fejér methods for linear and convex inequalities 79**

1 The basic construction of $M$-Fejér mappings for application to finite systems of linear inequalities . . . . . 79

2 Fejér processes with variable coefficient of relaxation . . . 80
2.1 The main theorem . . . 81
2.2 Fejér process with the mirror relaxation . . . 82

3 Application of Fejér processes to a system of convex inequalities . . . 84
3.1 Systems of inequalities in $\mathbb{R}^n$ . . . 84
3.2 Systems of inequalities in a Hilbert space . . . 86

4 Systems of convex inclusions . . . 87
4.1 General properties and constructions . . . 87
4.2 Consistent systems of inclusions . . . 89
4.3 Convergent processes for inconsistent systems of inclusions . . . 91
4.4 Cycles of immobility for inconsistent systems of convex inequalities . . . 93

5 On the rate of convergence of Fejér processes . . . 95

6 Fejér methods and nonsmooth optimization . . . 97
6.1 Problem of the saddle point of the Lagrange function . . . 98
6.2 Method of the exact penalty functions . . . 100
6.3 Fejér methods for the application to systems of convex inequalities and convex programming problems without assumptions of smoothness . . . 102
6.4 The basic process . . . 104

7 Aspects of stability of Fejér processes . . . 106

**Chapter IV Some topics of Fejér mappings and processes 109**

1 Decomposition and parallelization of Fejér processes . . . 109
1.1 Schemes of parallelization . . . 109
1.2 Schemes of parallelization for a linear programming problem . . . 112

2 Randomization of Fejér processes . . . 114
2.1 General theorems on convergence . . . 115
2.2 Some notes on the realization of the processes . . . 117

3 Fejér processes and inconsistent systems for linear inequalities . . . 119
3.1 Preliminary notes and information . . . 119

3.2 Fejér processes for problems of square approximation of inconsistent systems of linear inequalities . . . . . 121
3.3 Transition of results to the case of a system with additional constraints . . . . . 123

4 Fejér processes for improper problems of linear programming . . . . . 124
4.1 The basic approximative-Fejér process for improper linear programming problems of the first kind . . . 126
4.2 Approximative-Fejér process for improper problems of linear programming of the second kind 128
4.3 Fejér process for improper problems of linear programming of the third kind . . . . . 129

5 Normalized solutions of convex inequalities . . . . . 130
5.1 Auxiliary results . . . . . 131
5.2 Theorems on stability of the fixed points for quasi-contractions . . . . . 133
5.3 Iterative procedure for finding projection . . . . . 137

6 Fejér processes for inconsistent linear and convex inequality systems . . . . . 139

**Bibliography** **145**

**Notations** **151**

**Index** **153**