

Contents

Chapter 1

Fundamentals of the Theory of Difference Schemes

1.1. Basic Equations and their Adjoint 1

1.1.1. Norm Estimates of Certain Matrices 5

1.1.2. Computing the Spectral Bounds of a Positive Matrix 6

1.1.3. Eigenvalues and Eigenfunctions of the Laplace Operator 9

1.1.4. Eigenvalues and Eigenvectors of the Finite-Difference Analogue of the Laplace Operator 11

1.2. Approximation 15

1.3. Countable Stability 22

1.4. The Convergence Theorem 30

Chapter 2

Methods of Constructing Difference Schemes for Differential Equations

2.1. Method of Constructing Difference Equations for Problems with Discontinuous Coefficients on the Basis of an Integral Identity 35

2.2. Variational Methods in Mathematical Physics 42

2.2.1. The Ritz Method 43

2.2.2. The Galerkin Method 45

2.2.3. The Least-Squares Method 46

2.3. Difference Schemes for Equations with Discontinuous Coefficients Based on Variational Principles 47

2.3.1. Simple Difference Equations for a Diffusion Based on the Ritz Method 48

2.3.2. Constructions of Simple Difference Schemes Based on the Galerkin (Finite Elements) Method 51

2.4. General Approach to Variational-Difference Schemes for One-Dimensional Equations and Construction of Subspaces 53

2.5. Variational-Difference Schemes for Two-Dimensional Equations of Elliptic Type 57

2.5.1. The Ritz Method 57

2.5.2. The Galerkin Method 64

2.6. Variational Methods for Multidimensional Problems 67

2.6.1. Methods of Choosing the Subspaces 67

2.6.2. Coordinate-by-Coordinate Methods for Variational-Difference Schemes 69

2.7. Interpolation of Solutions of Difference Equations by Means of Splines 71

2.7.1. Interpolation of Functions of One Variable 72

2.7.2. Piece-Wise Interpolation with Smoothing 76

2.7.3. Interpolation of Functions of Two Variables 84

Chapter 3

Methods for Solving Stationary Problems of Mathematical Physics

- 3.1. Some Iterative Methods and their Optimization 88
 - 3.1.1. The Simple Iterative Method 90
 - 3.1.2. The Displacement Method 92
 - 3.1.3. The Chebyshev Acceleration Method 92
 - 3.1.4. The Over-Relaxation Method 97
 - 3.1.5. A Comparison of the Asymptotic Rate of Convergence for Various Iterative Methods 103
- 3.2. Gradient Iterative Methods 103
 - 3.2.1. The Residual Method 104
 - 3.2.2. The Two-Step Residual Method 106
 - 3.2.3. The Method of Conjugate Gradients 108
- 3.3. The Splitting-Up Method 113
- 3.4. The Splitting-Up Method with Variational Optimization 123
- 3.5. Equations with Singular Operators 126
- 3.6. Iterative Methods for Inaccurate Input Data 130
- 3.7. The Fast Fourier Transform 132
- 3.8. Factorization of Difference Equations 139

Chapter 4

Methods for Solving Non-Stationary Problems

- 4.1. Second-Order-Approximation Difference Schemes with Time-Varying Operators 142
- 4.2. Nonhomogeneous Equations of Evolution Type 145
- 4.3. Splitting-Up Methods for Nonstationary Problems 146
 - 4.3.1. The Stabilization Method 147
 - 4.3.2. The Predictor-Corrector Method 151
 - 4.3.3. The Component-by-Component Splitting-Up Method 154
 - 4.3.4. Some General Remarks 159
- 4.4. Multicomponent Splitting 160
 - 4.4.1. The Stabilization Method 160
 - 4.4.2. The Predictor-Corrector Method 162
 - 4.4.3. The Component-by-Component Splitting-Up Method Based on the Elementary Schemes 164
 - 4.4.4. Splitting-Up of Quasilinear Problems 169
- 4.5. General Approach to Component-by-Component Splitting 170
- 4.6. Methods of Solving Equations of Hyperbolic Type 174
 - 4.6.1. The Stabilization Method 174
 - 4.6.2. Reduction of the Wave Equation to an Evolution Problem 178

Chapter 5

Numerical Methods for Some Inverse Problems

- 5.1. Basic Definitions and Examples 185
- 5.2. Fourier Series Method for Inverse Evolution Problems 189
- 5.3. Inverse Evolution Problems with Time-Varying Operators 193
- 5.4. Methods of Perturbation Theory for Inverse Problems 199
 - 5.4.1. Some Problems of the Linear Theory of Measurements 199
 - 5.4.2. Conjugate Functions and the Notion of Value 200
 - 5.4.3. Perturbation Theory for Linear Functionals 203
 - 5.4.4. Numerical Methods for Inverse Problems and Design of Experiment 205

Chapter 6

The Simplest Problems of Mathematical Physics

- 6.1. The Poisson Equation 211
 - 6.1.1. The Dirichlet Problem for the One-Dimensional Poisson Equation 211
 - 6.1.2. The One-Dimensional Neumann Problem 213
 - 6.1.3. The Two-Dimensional Poisson Equation 215
 - 6.1.4. A Problem of Boundary Conditions 222
- 6.2. The Heat Equation 224
 - 6.2.1. The One-Dimensional Problem of Heat Conduction 224
 - 6.2.2. The Two-Dimensional Problem of Heat Conduction 229
- 6.3. The Wave Equation 230
- 6.4. The Equation of “Motion”
 - 6.4.1. The Simplest Equations of Motion 234
 - 6.4.2. The Two-Dimensional Equation of Motion with Variable Coefficients 241
 - 6.4.3. The Multidimensional Equation of Motion 246
- 6.5. On Increasing the Order of Approximation of Difference Schemes 251

Chapter 7

Numerical Methods in the Theory of Radiative Transfer

- 7.1. Problem Statement 259
- 7.2. The Transport Equation in Various Geometries 261
- 7.3. Numerical Solution of the Transport Equation in the Parallel-Plane Geometry 263
- 7.4. The Stationary Transport Problem 272
- 7.5. Nonisotropic Particle Scattering 276

Chapter 8

A Review of the Methods of Numerical Mathematics

- 8.1. The Theory of Approximation, Stability, and Convergence of Difference Schemes 279
- 8.2. Numerical Methods for Problems of Mathematical Physics 281
 - 8.2.1. Constructions of Difference Schemes 282
 - 8.2.2. Variational Methods 282
 - 8.2.3. Multidimensional Stationary Problems 283
 - 8.2.4. Multidimensional Nonstationary Problems 284
- 8.3. Conditionally Well-Posed Problems 287
- 8.4. Numerical Methods in Linear Algebra 288
 - 8.4.1. Direct Methods of Linear Algebra 288
 - 8.4.2. Iterative Methods 289
 - 8.4.3. Round-Off Error Analysis 291
 - 8.4.4. Complexes of Standard Programs 291
- 8.5. Optimization Problems in Numerical Mathematics 291
- 8.6. Some Trends in Numerical Mathematics 293

References 295

Index 315