Contents

Introduction — V

1 Function spaces — 1

1.1 Prerequisites and notation — **1**
Locally convex topological vector spaces — **1**
Linear operators — **2**
The dual space — **2**
Polar topologies — **3**
σ-rings — **4**
Vector-valued functions — **5**
Extended number systems — **5**
Semicontinuous functions — **6**
Convex sets — **6**

1.2 Spaces of vector-valued functions that vanish at infinity — **6**
Neighborhood functions — **7**
Function space neighborhoods — **7**
Function space neighborhood systems — **8**
Functions that vanish at infinity — **8**
Weakly lower continuous neighborhood functions — **8**
The function spaces $\mathcal{C}_{\mathfrak{W}}(X, E)$ — **10**
Examples — **12**
Function subspaces — **19**

1.3 Compactness in spaces of linear operators — **19**
Bounded operators — **20**
Compact and weakly compact operators — **23**
Examples — **28**

1.4 Vector space extensions — **29**
Locally convex cones — **30**
The dual cone — **31**
Sub and superlinear functionals — **32**
Convex sets and sublinear functionals — **33**
The locally convex cone $\mathcal{P}_E$ — **36**
The locally convex cone $\mathcal{Q}_F$ — **37**
Elements of $\mathcal{Q}_F$ representing convex sets — **42**
Examples — **42**
The cone $\mathcal{F}(X, \mathrm{Conv}(E))$ — **45**

2 Integration — 47

2.1 Operator-valued measures — **48**
Definition of an operator-valued measure — **48**

Bibliografische Informationen
https://d-nb.info/1294440519

digitalisiert durch DEUTSCHE NATIONAL BIBLIOTHEK

$\mathfrak{L}(E, F^{*\blacksquare})$-valued measures — **48**
Modulus of a measure — **50**
Bounded measures — **51**
Variation of a measure — **53**
Measure of bounded variation — **56**
Examples — **56**
Compact and weakly compact measures — **60**
Restriction of a measure — **60**
Sets of measure zero — **60**
Properties that hold almost everywhere — **61**
Point evaluation measures — **62**
Regularity of measures — **62**
Composition measures — **67**
2.2 Measurable functions — **68**
Set-valued step functions — **68**
Measurable vector-valued functions — **70**
Measurable set-valued functions — **73**
Examples — **83**
Composition of measurable functions — **85**
2.3 Integrals of set-valued functions — **86**
Integrals of step functions — **87**
Integrals of measurable functions — **88**
Integrals of set-valued functions — **101**
Integrals over sets in $\mathfrak{R}$ — **102**
Integrals over sets in $\mathfrak{A}$ — **103**
Integrals of vector-valued functions — **111**
Examples — **115**
Integrals with respect to composition measures — **117**
2.4 The convergence theorems — **123**
The relative topologies — **129**
Sequences of set-valued functions — **130**
The convergence theorem — **133**
Examples — **137**
2.5 Measures as linear operators — **139**
Properties of operators defined by an integral — **142**
Properties of measures representing an operator — **146**
Examples — **148**
2.6 Integral representation — **149**
The representation theorem — **149**
Positive operators — **156**
Lattice homomorphisms — **157**
Algebra homomorphisms — **158**

The spectral theorem — **160**
Compositions of operators with real- or complex-valued functions — **161**
Point evaluation operators — **163**
Examples and Remarks — **164**

3 Choquet theory — 168
3.1 $C(X)$-convex sets and functionals — **168**
$\mathcal{C}(X)$-convex sets — **172**
Ω-extremal point evaluations — **175**
Elementary operators — **185**
Examples — **187**
3.2 A Choquet ordering for linear operators on function spaces — **189**
Choquet cone — **189**
Choquet ordering — **190**
Upper and lower envelopes — **191**
Minimal elements — **196**
The Choquet boundary — **197**
$\mathfrak{C}$-superharmonic sets — **204**
3.3 Special cases and examples — **214**
The case that X is compact and that $E = F = \mathbb{R}$ — **214**
Sample settings in the general case — **215**
The case that $F = \mathbb{R}$ or $F = \mathbb{C}$ — **223**
The case that $E = \mathbb{R}$ or $E = \mathbb{C}$ — **224**
Examples — **227**
The case that both E and F are either $\mathbb{R}$ or $\mathbb{C}$ — **229**
Examples — **238**
The case that both E and F are ordered topological vector spaces — **240**
The case that both E and F are topological vector lattices — **242**

List of Symbols — 243

Bibliography — 245
Further Reading — **247**

Index — 249