

Crystal Optics

1.1 Introduction

Crystal optics is the branch of optics that describes the behavior of electromagnetic waves in anisotropic media, that is, media (such as crystals) in which light behaves differently depending on the direction in which the light is propagating. The characteristic phenomena of crystals that are studied in crystal optics include double refraction (birefringence), polarization of light, rotation of the plane of polarization, etc.

The phenomenon of double refraction was first observed in crystals of Iceland spar by the Danish scientist E. Bartholin in 1669. This date is considered the beginning of crystal optics. Problems of the absorption and emission of light by crystals are studied in crystal spectroscopy. The effect of electric and magnetic fields, mechanical stress, and ultrasound waves on the optical properties of crystals are studied in electro-optics, magneto-optics, photoelasticity, acousto-optics, and photorefractivity, which are based on the fundamental laws of crystal optics.

Since the lattice constant (of the order of $10 \text{ \AA}$) is much smaller than the wavelength of visible light ($4000\text{--}7000 \text{ \AA}$), a crystal may be regarded as a homogeneous but anisotropic medium. The optical anisotropy of crystals is caused by the anisotropy of the force field of particle interaction. The nature of the field is related to crystal symmetry. All crystals, except crystals of the cubic system, are optically anisotropic.

1.2 Index Ellipsoid or Optical Indicatrix

In isotropic materials, the electric field displacement vector $\mathbf{D}$ is parallel to the electric field vector $\mathbf{E}$, related by $\mathbf{D} = \epsilon_0 \epsilon_r \mathbf{E} = \epsilon_0 \mathbf{E} + \mathbf{P}$, where ϵ_0 is the permittivity of free space, ϵ_r is the unitless relative dielectric constant, and $\mathbf{P}$ is the material polarization vector.

The optical anisotropy of transparent crystals is due to the anisotropy of the dielectric constant. In an anisotropic dielectric medium (a crystal, for example), the vectors $\mathbf{D}$ and $\mathbf{E}$ are no longer parallel; each component of the electric

flux density $\mathbf{D}$ is a linear combination of the three components of the electric field E .

$$D_i = \sum_j \epsilon_{ij} E_j \quad (1.1)$$

where $i, j = 1, 2, 3$ indicate the x, y , and z components, respectively. The dielectric properties of the medium are therefore characterized by a 3×3 array of nine coefficients $\{\epsilon_{ij}\}$ forming a tensor of second rank known as the electric permittivity tensor and denoted by the symbol ϵ . Equation (1.1) is usually written in the symbolic form $\mathbf{D} = \epsilon \mathbf{E}$. The electric permittivity tensor is symmetrical, $\epsilon_{ij} = \epsilon_{ji}$, and is therefore characterized by only six independent numbers. For crystals of certain symmetries, some of these six coefficients vanish and some are related, so that even fewer coefficients are necessary.

Elements of the permittivity tensor depend on the choice of the coordinate system relative to the crystal structure. A coordinate system can always be found for which the off-diagonal elements of ϵ_{ij} vanish, so that

$$D_1 = \epsilon_1 E_1, \quad D_2 = \epsilon_2 E_2, \quad D_3 = \epsilon_3 E_3 \quad (1.2)$$

where $\epsilon_1 = \epsilon_{11}$, $\epsilon_2 = \epsilon_{22}$, and $\epsilon_3 = \epsilon_{33}$. These are the directions for which E and D are parallel. For example, if E points in the x -direction, D must also point in the x -direction. This coordinate system defines the principal axes and principal planes of the crystal. The permittivities ϵ_1 , ϵ_2 , and ϵ_3 correspond to refractive indices.

$$n_1 = \left(\frac{\epsilon_1}{\epsilon_0} \right)^{1/2}, \quad n_2 = \left(\frac{\epsilon_2}{\epsilon_0} \right)^{1/2}, \quad n_3 = \left(\frac{\epsilon_3}{\epsilon_0} \right)^{1/2} \quad (1.3)$$

are known as the principal refractive indices and ϵ_0 is the permittivity of free space.

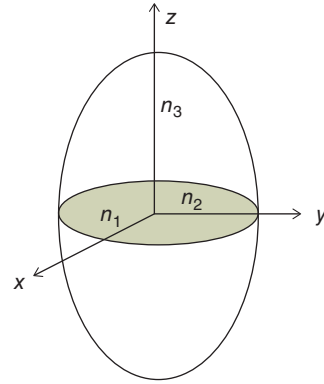
In crystals with certain symmetries two of the refractive indices are equal ($n_1 = n_2 \neq n_3$) and the crystals are called uniaxial crystals. The indices are usually denoted $n_1 = n_2 = n_o$ and $n_3 = n_e$. The uniaxial crystal exhibits two refractive indices, an “ordinary” index (n_o) for light polarized in the x - or y -direction, and an “extraordinary” index (n_e) for polarization in the z -direction. The crystal is said to be positive uniaxial if $n_e > n_o$ and negative uniaxial if $n_e < n_o$. The z -axis of a uniaxial crystal is called the optic axis. In other crystals (those with cubic unit cells, for example) the three indices are equal and the medium is optically isotropic. Media for which the three principal indices are different (i.e. $n_1 \neq n_2 \neq n_3$) are called biaxial. Light polarized at some angle to the axes will experience a different phase velocity for different polarization components and cannot be described by a single index of refraction. This is often depicted as an index ellipsoid.

The optical properties of crystals are described by the index ellipsoid or optical indicatrix. It is generated by the equation

$$\frac{x_1^2}{n_1^2} + \frac{x_2^2}{n_2^2} + \frac{x_3^2}{n_3^2} = 1 \quad (1.4)$$

where x_1, x_2 , and x_3 are the principal axes of the dielectric constant tensor and n_1, n_2 , and n_3 are the principal dielectric constants, respectively. Figure 1.1 shows the

Figure 1.1 The index ellipsoid. The coordinates (x, y, z) are the principal axes and (n_1, n_2, n_3) are the principal refractive indices of the crystal.



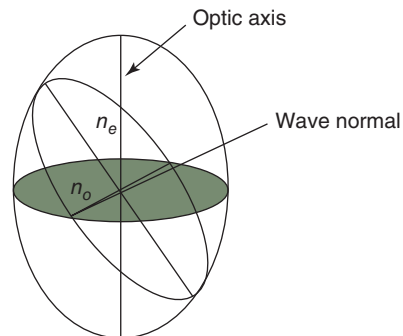
optical indicatrix of a biaxial crystal. It is a general ellipsoid with $n_1 \neq n_2 \neq n_3$ representative of the optical properties of triclinic, monoclinic, and orthorhombic crystals.

1.3 Effect of Crystal Symmetry

In the case of cubic crystals, which are optically isotropic, ϵ is independent of direction and the optical indicatrix becomes a sphere with radius n . In crystals of intermediate systems (trigonal, tetragonal, and hexagonal), the indicatrix is necessarily an ellipsoid of revolution about the principal symmetry axis (Figure 1.2). The central section is perpendicular to the principal axis, and only this central section is a circle of radius n_o . Hence, only for a wave normal along the principal axis is there no double refraction. The principal axis is called the optic axis and the crystals are said to be uniaxial. A uniaxial crystal is called optically positive (+) when $n_e > n_o$ and negative (−) when $n_e < n_o$.

For crystals of the lower systems (orthorhombic, monoclinic, and triclinic), the indicatrix is a triaxial ellipsoid. There are two circular sections (Figure 1.3) and hence two privileged wave normal directions for which there is no double refraction. These two directions are called the primary optic axes or simply the optic axes, and the crystals are said to be biaxial.

Figure 1.2 The indicatrix for a (positive) uniaxial crystal.



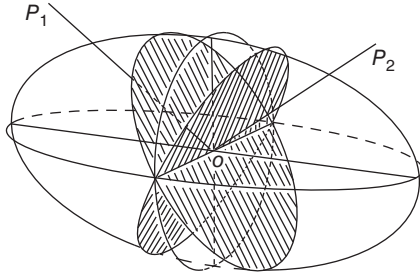


Figure 1.3 The two circular sections of the indicatrix and the two primary optic axes OP_1 , OP_2 for a biaxial crystal.

1.4 Wave Surface

If a point source of light is situated within a crystal the wave front emitted at any instant forms a continuously expanding surface. The geometric locus of points at a distance v from a point O is called the ray surface or wave surface. Actually, the wave surface is a wave front (or pair of wave fronts) completely surrounding a point source of monochromatic light. This is also a double-sheeted surface. In most crystalline substances, however, two wave surfaces are formed; one is called the ordinary wave surface and the other is called the extraordinary wave surface. In both positive and negative crystals, the ordinary wave surface is a sphere and the extraordinary wave surface is an ellipsoid of revolution.

1.4.1 Uniaxial Crystal

In uniaxial crystals, one surface is a sphere and the other is an ellipsoid of revolution touching one another along the optical axis – OZ as shown in Figure 1.4a,b. In positive (+) crystals ($n_e > n_o$) the ellipsoid is inscribed within the sphere (Figure 1.4a), whereas in negative (–) crystals ($n_e < n_o$) the sphere is inscribed within the ellipsoid (Figure 1.4b).

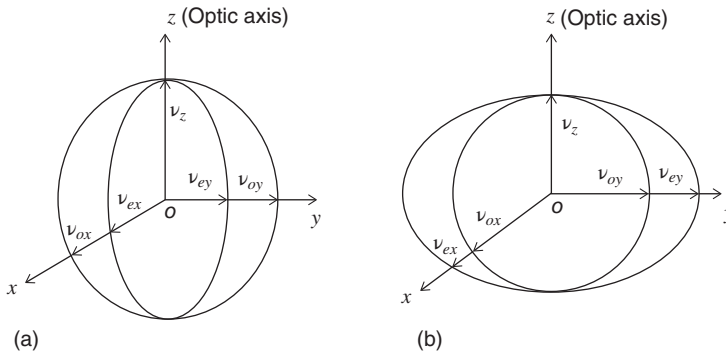


Figure 1.4 Ray surfaces of uniaxial crystals: (a) positive, (b) negative, (OZ) optical axis of the crystal, (v_o) and (v_e) phase velocities of ordinary and extraordinary waves propagating in the crystals.

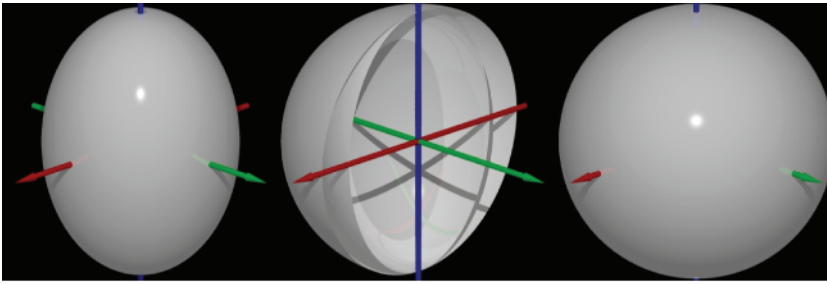


Figure 1.5 Positive uniaxial medium wave surface. Inner fold (left), vertical section (center), and outer fold (right). Source: Latorre et al. 2012 [1]. Reproduced with permission of Springer Nature.

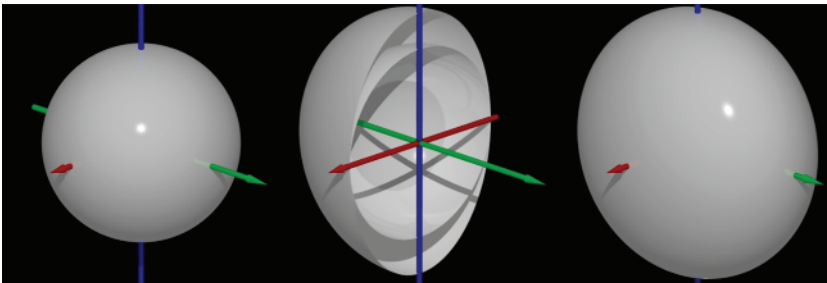


Figure 1.6 Negative uniaxial medium wave surface. Inner fold (left), vertical section (center), and outer fold (right). Source: Latorre et al. 2012 [1]. Reproduced with permission of Springer Nature.

The dependence of the ray velocity of a plane wave propagating in a crystal on the direction of propagation and the nature of polarization of the wave leads to the splitting of light rays in crystals. In a uniaxial crystal, one of the refracted rays obeys the usual laws of refraction and is therefore called the ordinary ray, whereas the other ray does not (it does not lie in the plane of incidence) and is called the extraordinary ray. The three-dimensional view of positive and negative uniaxial medium wave surfaces can be seen in Figures 1.5 and 1.6.

1.4.2 Biaxial Crystal

It is very difficult to imagine what shapes the biaxial wave surface will have. For wave normals that lie in any of the three principal planes of the indicatrix the situation is very similar to that described for the uniaxial crystal. Three cross-sectional views of the wave surfaces for a biaxial crystal are given in Figure 1.7.

In biaxial media, the wave surface equations are fourth-order polynomials with even powers only; that is, the surface is symmetrical with respect to the origin. Both surface folds intersect only at four symmetrical points, as can be seen in Figure 1.8. Note that this intersection does not yield a curve but only the four points.

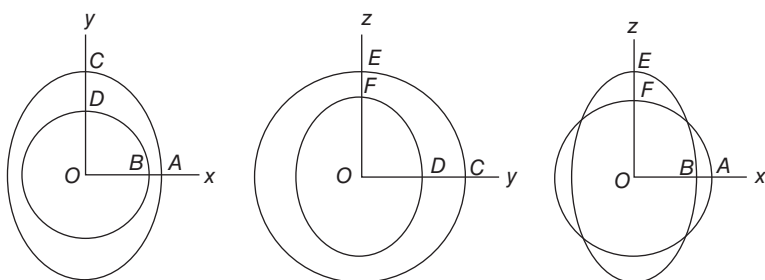


Figure 1.7 Principal sections of the wave surface for a biaxial crystal.

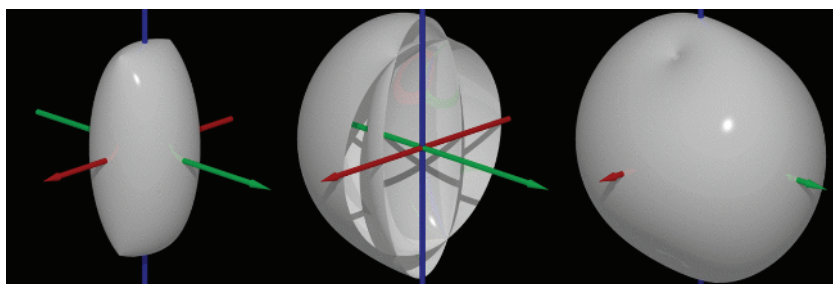


Figure 1.8 Wave surface for a biaxial medium. Inner fold (left), vertical section (center), and outer fold (right). Source: Latorre et al. 2012 [1]. Reproduced with permission of Springer Nature.

1.5 Birefringence

When a beam of nonpolarized light passes into a calcite or quartz crystal, the light is decomposed into two beams that refract at different angles. This phenomenon is called birefringence or double refraction. The ray for which Snell's law holds is called the ordinary or O-ray, and the other is called the extraordinary or E-ray (Figure 1.9).

Birefringent materials are optically anisotropic (their properties depend on the direction a light beam takes across them) because their molecules do not respond to the incident light evenly in all directions. This arises from their molecular (bond strengths) and crystal (arrangement) structures. However, in all such materials there is at least one optic axis (some materials have two) along which propagating light can travel with no consequences to either (any) component of its electric vector. This axis serves as a kind of reference. Light traveling in any other direction through the crystal experiences two different refractive indices and is split into components that travel at different speeds and have perpendicular polarizations.

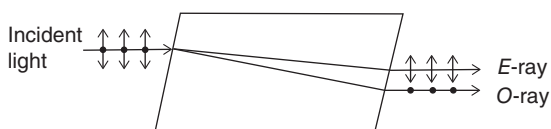
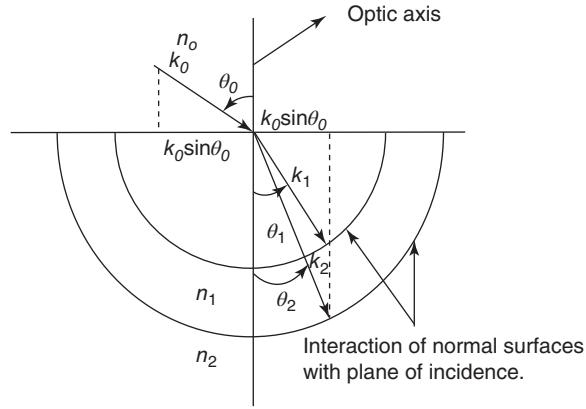


Figure 1.9 Side view of the double refraction of light by a calcite crystal.

Figure 1.10 Double refraction at the boundary of an anisotropic medium.



This effect of double refraction or birefringence is further demonstrated in Figure 1.10 [2]. In Figure 1.10, subscript 0 indicates the incident wave, while 1 and 2 indicate the refracted waves. The refractive index for ordinary wave is denoted by n_o and is independent of the direction of propagation. The refractive index for extraordinary wave is denoted by $n_e(\theta)$ and depends on the direction of propagation (θ) relative to the optic axis.

The behavior of refractive index is usually described in terms of the refractive index surface, i.e. the index ellipsoid. In the case of the ordinary ray it is a sphere, while for the extraordinary ray it is an ellipsoid. That is, in terms of ellipsoid, this effect becomes a three-dimensional body with cylindrical symmetry. The two indices of refraction are then identical ($n_x = n_y$), so that the plane intersecting perpendicular to the optical axis forms a circle. If z -axis is considered as the axis of cylindrical symmetry (the optical axis of a uniaxial crystal), then for uniaxial crystal, the principal indices of refraction are

$$n_o^2 = \frac{\epsilon_x}{\epsilon_0} = \frac{\epsilon_y}{\epsilon_0} \quad \text{and} \quad n_e^2 = \frac{\epsilon_z}{\epsilon_0} \quad (1.5)$$

where ϵ_0 is the dielectric constant in free space ($\sim 8.85 \times 10^{-12}$ F/m); ϵ_x , ϵ_y , and ϵ_z are the dielectric constants along x , y , and z -axes. For uniaxial crystals $\epsilon_x = \epsilon_y$. It is also a known fact that refractive index and critical angle of materials are related by

$$\sin c_o = \frac{1}{n_o} \quad \text{and} \quad \sin c_e = \frac{1}{n_e} \quad (1.6)$$

where c_o and c_e are the critical angles at the ordinary and extraordinary axes respectively. The critical angle of a material determines whether an internal ray will be reflected back into the material. As shown in Eq. (1.6), it is a function of the refractive index, and hence, the higher the refractive index the lower the critical angle.

In Figure 1.11a, the incident light rays giving rise to the ordinary and extraordinary rays enter the crystal in a direction that is oblique with respect to the optical axis and are responsible for the observed birefringent character. When an incident ray enters the crystal perpendicular to the optical axis, it is separated into ordinary and extraordinary rays, but instead of taking different pathways, the

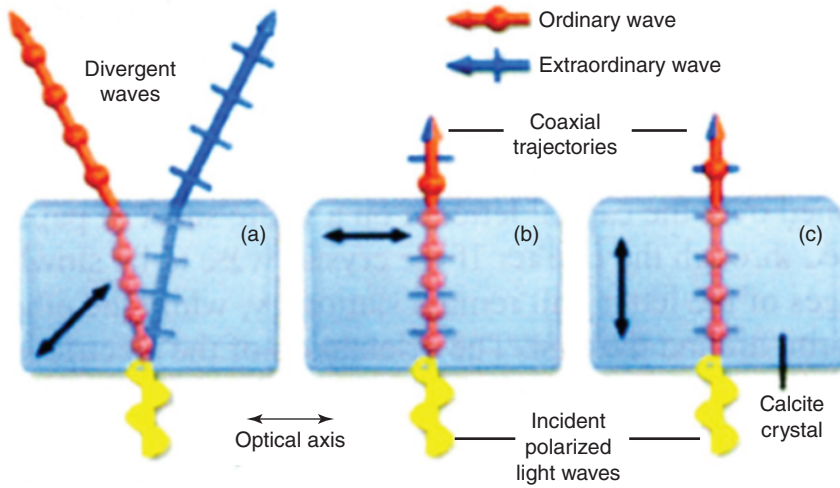


Figure 1.11 Separation of light waves by a birefringent crystal [3]. Source: Courtesy of Nikon.

trajectories of these rays are coincident. Even though the ordinary and extraordinary rays emerge from the crystal at the same location, they exhibit different optical path lengths and are subsequently shifted in phase relative to one another (Figure 1.11b). In the case where incident light rays impact the crystal in a direction that is parallel to the optical axis (Figure 1.11c), they behave as ordinary light rays and are not separated into individual components by an anisotropic birefringent crystal. Calcite and other anisotropic crystals act as if they were isotropic materials (such as glass) under these circumstances. The optical path lengths of the light rays emerging from the crystal are identical, and there is no relative phase shift.

1.6 Polarization of Light

Polarization generally just means “orientation.” It comes from the Greek word *polos*, for the axis of a spinning globe. Wave polarization occurs for *vector* fields. For light (electromagnetic waves) the vectors are the electric and magnetic fields, and the light’s polarization direction is by convention along the direction of the electric field. Generally, we should expect fields to have three vector components, e.g. (x, y, z) , but light waves only have two non-vanishing components: the two that are perpendicular to the direction of the wave.

Electromagnetic waves are the solutions of Maxwell’s equations in a vacuum:

$$\begin{aligned}
 \nabla \times \mathbf{E} &= 0 \\
 \nabla \times \mathbf{B} &= 0 \\
 \nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} \\
 \nabla \times \mathbf{B} &= \epsilon_0 \mu_0 \frac{\partial \mathbf{E}}{\partial t}
 \end{aligned} \tag{1.7}$$

Figure 1.12 Schematic view of electromagnetic wave propagation along the z -axis.

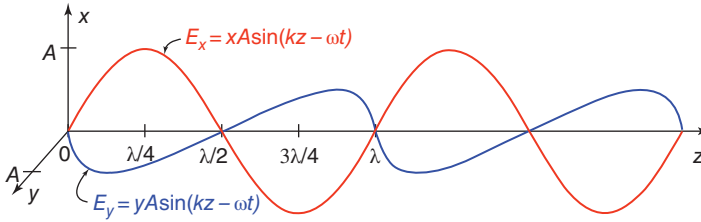
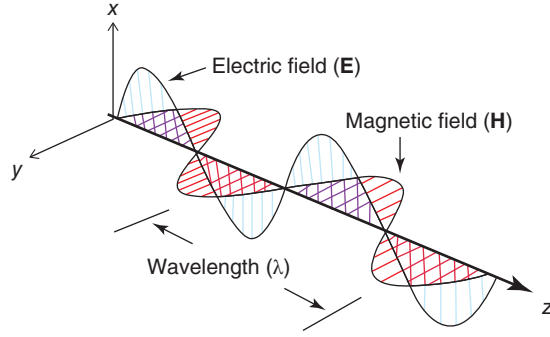


Figure 1.13 Propagation of E_x and E_y components along the z -axis [4]. Source: Courtesy of Semrock.

In order to satisfy all four equations, the waves must have the $\mathbf{E}$ and $\mathbf{B}$ fields transverse to the propagation direction. Thus, if the wave is traveling along the positive z -axis, the electric field can be parallel to the x -axis and $\mathbf{B}$ -field parallel to the y -axis (Figure 1.12).

We shall call the two distinct waves E_x and E_y , where we denote these by vectors to remind that they point in (or oscillate along) a certain direction (the x - and y -directions, respectively) as shown in Figure 1.13. The amplitude of the light wave describes how the wave propagates in position and time. Mathematically, we can write it as a “sine wave” where the angle of the sine function is a linear combination of both position and time terms:

$$E(x, t) = A \sin \left(2\pi \frac{z}{\lambda} \pm 2\pi \nu t \right) \quad (1.8)$$

where A is called the “amplitude factor,” the variable λ is the wavelength, and the variable ν is the frequency. If a snapshot of the wave could be taken at a fixed time, λ would be the distance from one wave peak to the next. If one sits at a fixed point in space and counts the wave peaks as they pass by, ν gives the frequency of these counts, or $1/\nu$ gives the time between peaks. The sign between the position and time terms determines the direction the wave travels: when the two terms have the opposite sign (i.e. the “ $-$ ” sign is chosen), the wave travels in the positive z -direction.

For convenience, we often use two new variables called the wave number $k = 2\pi/\lambda$ and the angular frequency $\omega = 2\pi\nu$, which absorb the factor of 2π , so that the wave amplitude can now be written more compactly as

$$E(x, t) = A \sin(kz \pm \omega t) \quad (1.9)$$

Using this description of a single transverse orientation of a light wave, we can now consider multiple orientations to describe different states of polarization.

1.6.1 Linear Polarization – Equal Amplitudes

Here is an example of two waves E_x and E_y viewed in a “fixed time” picture (say at $t = 0$). The amplitude E or the potential for a charged particle to feel a force is vibrating along both the x - and y -directions. An actual charged particle would feel both of these fields simultaneously, or it would feel

$$E = E_x + E_y = (x + y)A \sin(kz - \omega t) \quad (1.10)$$

If we look down the propagation axis in the positive z -direction, the vector $\mathbf{E}$ at various locations (and at $t = 0$) appears as in Figure 1.14.

That is, $\mathbf{E}$ appears to oscillate along a line oriented at 45° with respect to the x -axis. Hence this situation is called linear polarization.

Equivalently, we could view the wave at a particular location (“fixed position”) and watch its amplitude evolve with time. Suppose we sit at the position $z = 0$. Then we see that

$$E = E_x + E_y = -(x + y)A \sin(2\pi\nu t) \quad (1.11)$$

which appears as in Figure 1.15.

1.6.2 Linear Polarization – Unequal Amplitudes

If the two components E_x and E_y have unequal amplitude factors, we can see that the light wave is still linearly polarized (see Figure 1.16).

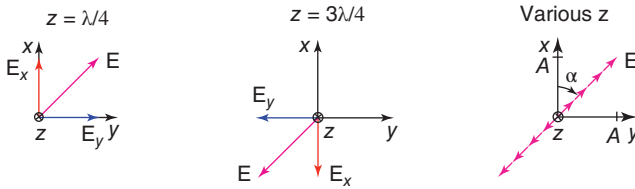


Figure 1.14 Orientation of $\mathbf{E}$ vector at various locations along the z -axis [4]. Source: Courtesy of Semrock.

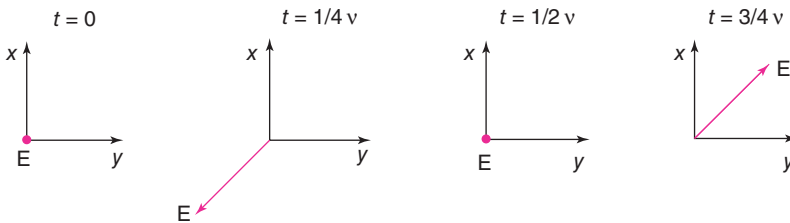


Figure 1.15 The oscillation of $\mathbf{E}$ vector back and forth along the same 45° line as time evolves [4]. Source: Courtesy of Semrock.

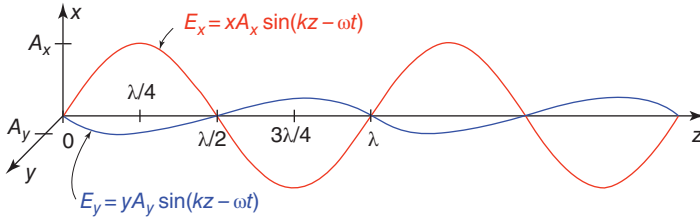
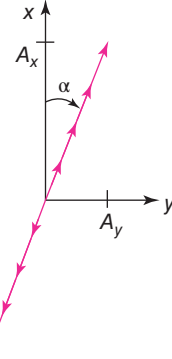


Figure 1.16 Propagation of the unequal E_x and E_y components along the z -axis [4]. Source: Courtesy of Semrock.

Figure 1.17 Orientation of $\mathbf{E}$ vector at various locations along the z -axis (when $A_x \neq A_y$) [4]. Source: Courtesy of Semrock.



If $A_x \neq A_y$, the total wave E is linearly polarized, but it is no longer oriented at 45° with respect to the x -axis. In fact, we can see that it is oriented at an angle α where

$$\tan \alpha = \frac{A_y}{A_x} \quad (1.12)$$

In other words, if we look down the propagation axis in the positive x -direction, the vector $\mathbf{E}$ at various locations (and at $t = 0$) appears as in Figure 1.17.

1.6.3 Circular Polarization

Suppose the two components have equal amplitudes again, but now consider the case where these two components are not in phase, such that the angles of the “sine functions” are different. In particular, suppose there is a constant phase difference of $\pi/2$ between them, which corresponds to a distance of $\lambda/4$ in the “fixed time” picture.

The x -component is

$$E_x = xA \sin \left(kz - \omega t - \frac{\pi}{2} \right) \quad (1.13)$$

while the y -component is as before

$$E_y = yA \sin(kz - \omega t) \quad (1.14)$$

This case appears as in Figure 1.18.

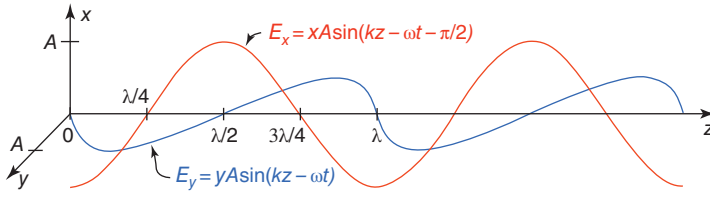


Figure 1.18 Propagation of E_x and E_y components along the z -axis when they are at a constant phase difference of $\pi/2$ [4]. Source: Courtesy of Semrock.

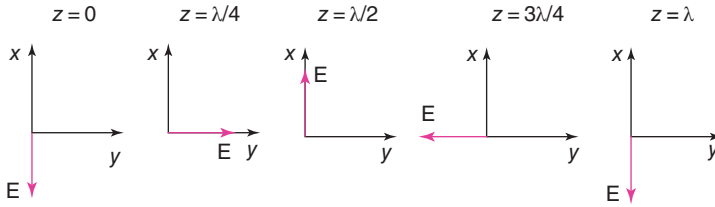


Figure 1.19 Orientation of E vector in the x - y plane at a fixed time [4]. Source: Courtesy of Semrock.

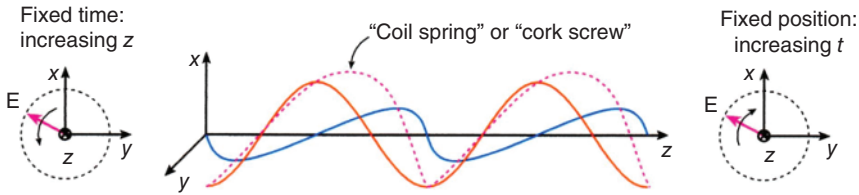


Figure 1.20 Circular orientation of E vector along the z -axis [4]. Source: Courtesy of Semrock.

To understand what is going on physically, again look down the z -axis (at time $t = 0$). We can see that the tip of E traces out a circle as we follow the wave along the z -axis at a fixed time (Figure 1.19).

Similarly, if we sit at a fixed position, the tip of E appears to trace out a circle as time evolves. Hence this type of polarization is called circular polarization (Figure 1.20).

1.6.4 Elliptical Polarization

All the states of polarization described above are actually special cases of the most general state of polarization, called elliptical polarization, in which the tip of the electric field vector E traces out an ellipse in the x - y plane. The two components might have unequal amplitudes $A_x \neq A_y$, and also might contain a different relative phase, often denoted δ . That is, we may write generally the x -component as follows:

$$E_x = xA_x \sin(kz - \omega t + \delta) \quad (1.15)$$

while the y -component is as before:

$$E_y = yA_y \sin(kz - \omega t) \quad (1.16)$$

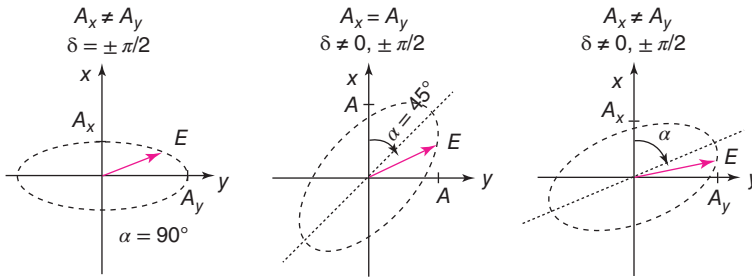


Figure 1.21 Elliptical polarization of E vector in the x - y plane [4]. Source: Courtesy of Semrock.

where, as before, $E = E_x + E_y$. The three special cases described in Sections 1.1, 1.2, and 1.3 thus correspond to $A_x = A_y$ and $\delta = 0$ (linear polarization; equal amplitudes); $A_x \neq A_y$ and $\delta = 0$ (linear polarization; unequal amplitudes); and $A_x = A_y$ with $\delta = -\pi/2$ (circular polarization), respectively. Some other examples of more general states of elliptical polarization are shown below (Figure 1.21).

1.7 Changing the Polarization of Light

Unpolarized light can be polarized using a polarizer or polarizing beam splitter (PBS) and the state of already polarized light can be altered using a polarizer and/or optical components that are phase retarders. In this section, we explore some examples of these types of components.

1.7.1 Polarizer and Polarizing Beam Splitters

A polarizer transmits only a single orientation of linear polarization and blocks the rest of the light. For example, a polarizer oriented along the x -direction passes E_x and blocks E_y (see Figure 1.22).

Some polarizer eliminates the non-passed polarization component (E_y in the above example) by absorbing it, while others reflect this component. Absorbing polarizers are convenient when it is desirable to completely eliminate one polarization component from the system. A disadvantage of absorbing polarizers is that they are not very durable and may be damaged by high-intensity light (as found in many laser applications).

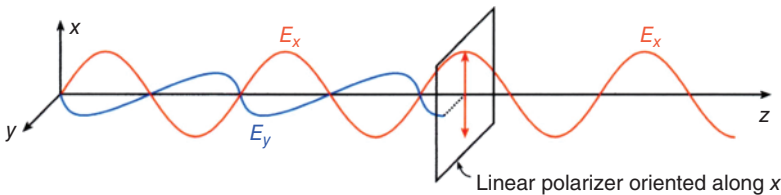


Figure 1.22 Linear polarizer transmits E_x component of the light oriented along the x -axis [4]. Source: Courtesy of Semrock.

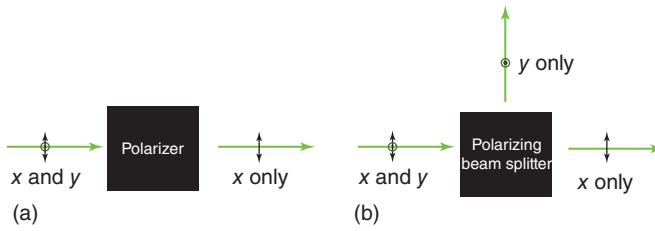


Figure 1.23 Polarization of light (y -component) by the absorbing polarizer (a), and by the reflective beam splitter (b) [4]. Source: Courtesy of Semrock.

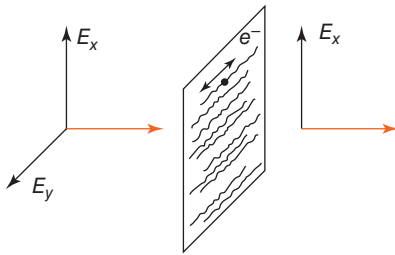


Figure 1.24 Transmission of E_x component of light through the PVA sheet Polaroid [4]. Source: Courtesy of Semrock.

When a reflective polarizer is operated in such a way that the blocked (i.e. reflected) polarization component is deflected into a convenient direction, such as 90° relative to the transmitted polarization component, then the polarizer acts like a PBS, as shown in Figure 1.23. Most PBSs are very efficient polarizers for the transmitted light (i.e. the ratio of desired to undesired polarization is very high); however, the reflected light generally contains some of both polarization components.

There are different ways of making a polarizer. However, as an example, consider one of the most popular absorbing polarizers: the well-known Polaroid “H-Sheet.” This polarizer, invented by E. H. Land in 1938, is a plastic, poly-vinyl alcohol (PVA) sheet that has been heated and then stretched in one direction, forming long, nearly parallel hydrocarbon molecule chains. After dipping the sheet into an iodine-rich ink, long iodine chains form along the hydrocarbon molecules. Electrons freely move along the iodine chains, but do not easily move perpendicular to the chains. This ability for electrons to move freely in one direction but not the perpendicular direction is the key principle upon which most absorbing polarizers are based (see Figure 1.24).

When the electric field of a light wave encounters the sheet, the component parallel to the chains causes electrons to oscillate along the direction of that component (E_y in the above example), thus absorbing energy and inhibiting the component from passing through the sheet. Because electrons cannot respond to the other component (E_x), it is readily transmitted.

1.7.2 Birefringent Wave Plate

Some materials have a different index of refraction for light polarized along different directions. This phenomenon is called birefringence. For example, suppose that light polarized along the x -direction sees an index of n_x while light

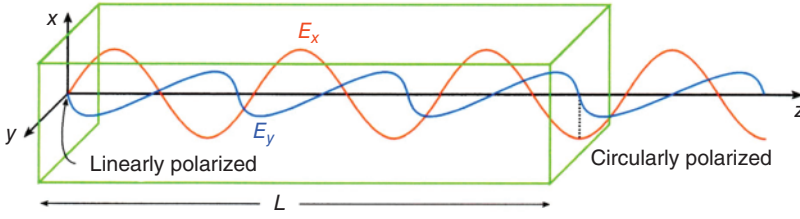


Figure 1.25 Transformation of linearly polarized light into circularly polarized light by the birefringent material [4]. Source: Courtesy of Semrock.

polarized along the y -direction sees an index n_y . Now, suppose linearly polarized light passes through a piece of such a birefringent material of length L , where the linear polarization axis is oriented at 45° angle with respect to the x - and y -axes. The fixed time picture thus looks as in Figure 1.25.

We can see that in general the light emerges in a different state of elliptic polarization. In fact, for the example illustrated above, the particular choice of L for a given difference between n_x and n_y causes the linearly polarized light at the input end to be converted to circularly polarized light at the other end of the birefringent material.

Consider the phases accumulated by the two component waves as they travel through the birefringent material. The waves can be described by

$$E_x = xA \sin\left(\frac{2\pi}{\lambda}n_x z - \omega t\right) \quad \text{and} \quad E_y = yA \sin\left(\frac{2\pi}{\lambda}n_y z - \omega t\right) \quad (1.17)$$

After traveling a length L , the waves have accumulated the respective phases of

$$\theta_x = \frac{2\pi}{\lambda}n_x L \quad \text{and} \quad \theta_y = \frac{2\pi}{\lambda}n_y L \quad (1.18)$$

If the difference between the two phase values is $\pi/2$, then the wave emerging from the material (say into air) will be circularly polarized. This occurs when

$$\theta_y - \theta_x = \frac{2\pi}{\lambda}(n_y - n_x)L = \frac{2\pi}{\lambda}\Delta n L = \frac{\pi}{2} \quad (1.19)$$

or when

$$\Delta n L = \frac{\lambda}{4} \quad (1.20)$$

Because of this relationship, a material with birefringence Δn of the appropriate thickness L to convert linear polarization to circular polarization is called a quarter-wave plate.

Some materials, especially crystals, are naturally anisotropic at microscopic (sub-wavelength) size scales. For example, calcite (CaCO_3) is shown in Figure 1.26. The structure, and hence the response to polarized light, along the c -direction is markedly different than that along the a - and b -directions, thus leading to a different index of refraction for light polarized along this direction (see Figure 1.26).

Other materials are nominally isotropic, but when they are bent or deformed in some way, they become anisotropic and therefore exhibit birefringence. This effect is widely used to study the mechanical properties of materials with optics.

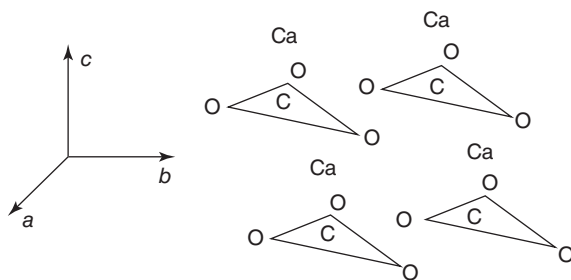


Figure 1.26 Structure of CaCO_3 along the a -, b -, and c -axis [4]. Source: Courtesy of Semrock.

1.8 Effects of Reflection and Transmission on Polarization

The polarization of light reflected and transmitted at an interface between two media or at a thin-film multilayer coating can be altered dramatically. These two cases are considered below.

1.8.1 Interface Between Two Media

When light is incident on an interface between two different media with different indexes of refraction, some of the light is reflected and some is transmitted. When the angle of incidence is not normal, different polarizations are reflected (and transmitted) by different amounts. This dependence was first properly described by Fresnel and hence it is often called “Fresnel reflection.” It is simplest to describe the polarization of the incident, the reflected, and transmitted (refracted) light in terms of a vector component perpendicular to the plane of incidence, called the “ s ” component, and a component parallel to the plane of incidence, called the “ p ” component. The “plane of incidence” is the plane that contains the incident ray and the transmitted and reflected rays (i.e. all of these rays lie on one plane). In the example in Figure 1.27, the plane of incidence is the plane containing the x - and z -axes. That is, $E_s \parallel y$, while E_p lies in the x - z plane.

The angle of the reflected ray, θ_r , is always equal to the angle of the incident ray, θ_i ; this result is called the “law of reflection.” The angle of the transmitted (or refracted) ray, θ_t , is related to the angle of incidence by the well-known “Snell’s law” relationship: $n_i \sin \theta_i = n_t \sin \theta_t$.

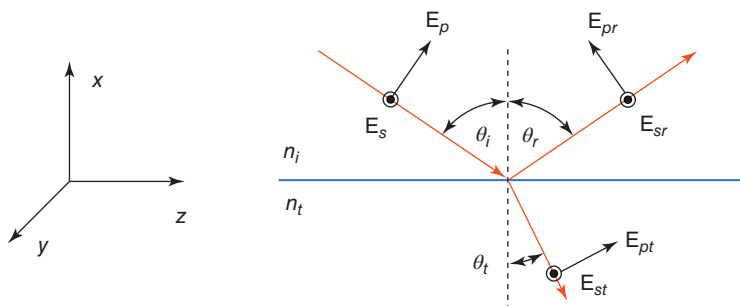


Figure 1.27 Polarization of the incident, reflected, and transmitted (refracted) light in terms of vector components (“ s ” and “ p ” components) [4]. Source: Courtesy of Semrock.

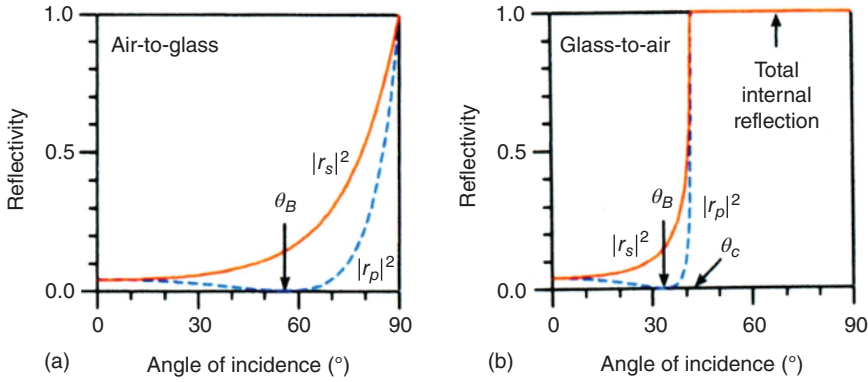


Figure 1.28 The intensity of reflection coefficient for a light wave for air-to-glass (a) and glass-to-air (b) [4]. Source: Courtesy of Semrock.

It turns out that s-polarized light is always more highly reflected than p-polarized light. In fact, at a special angle called “Brewster’s angle,” denoted θ_B , the p-polarized component sees no reflection, or is completely transmitted. Brewster’s angle is given by $\theta_B = \arctan(n_t/n_i)$. The power or intensity reflection coefficients for a light wave (i.e. the squares of the amplitude reflection coefficients) for air-to-glass and glass-to-air polarization appear as in Figure 1.28a and b, respectively.

The Fresnel reflection coefficients for non-normal incidence are given by the equations:

$$r_s = \frac{n_i \cos \theta_i - n_t \cos \theta_t}{n_i \cos \theta_i + n_t \cos \theta_t} \quad \text{and} \quad r_p = \frac{n_t \cos \theta_i - n_i \cos \theta_t}{n_t \cos \theta_i + n_i \cos \theta_t} \quad (1.21)$$

Notice from the graph above on the right that for the case of reflection from a higher index region to a lower index region (in this case glass-to-air, or $n_i = 1.5$ and $n_t = 1.0$), the reflectivity becomes 100% for all angles greater than the “critical angle” $\theta_c = \arcsin(n_t/n_i)$ and for both polarizations. This phenomenon is known as “total internal reflection” (TIR).

For angles of incidence below the critical angle only the amplitudes of the different polarization components are affected by reflection or transmission at an interface. Except for discrete changes of π (or 180°), the phase of the light is unchanged. Thus, the state of polarization can change in only limited ways. For example, linearly polarized light remains linearly polarized, although its orientation (angle α) may rotate. However, for angles greater than θ_c , different polarizations experience different phase changes, and thus TIR can affect the state of polarization of a light wave in the same way birefringence does. Thus, linearly polarized light may become elliptical or vice versa, in addition to changes in the orientation.

1.8.2 Multilayer Thin-Film Filters

Multilayer thin-film coatings have a large number of interfaces, since they are generally comprised of alternating layers of high- and low-index layer materials. The fraction of incident light intensity I_{in} that is reflected (I_R) and transmitted

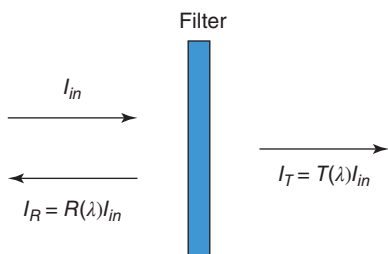


Figure 1.29 The fraction of intensity reflection (I_R) and transmission (I_T) for normal incidence of light on a multilayer thin film filter [4]. Source: Courtesy of Semrock.

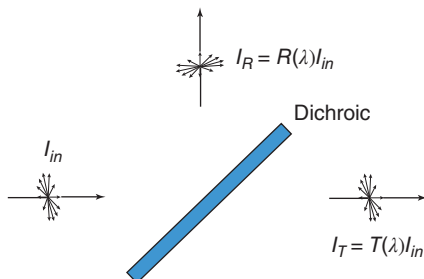


Figure 1.30 The fraction of intensity reflection (I_R) and transmission (I_T) for the non-normal, incoherent, and unpolarized incidence of light [4]. Source: Courtesy of Semrock.

(I_T) through a thin-film coating can be calculated from the indexes of refraction and the precise thicknesses of each layer. These intensity reflection and transmission functions $R(\lambda)$ and $T(\lambda)$, respectively, generally depend strongly on the wavelength of the light, because the total amount of light reflected from and transmitted through the coating comes from the interference of many individual waves that arise from the partial reflection and transmission at each interface. That is why optical filters based on thin-film coatings are called “interference filters” (see Figure 1.29).

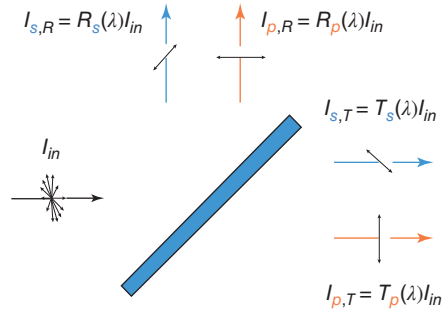
When an optical filter is used at a non-normal angle of incidence, as is common with the so-called “plate beam splitters,” the filter can impact the polarization of the light. If the incident light is incoherent and unpolarized and the optical system is “blind” to polarization, the standard intensity reflection and transmission functions $R(\lambda)$ and $T(\lambda)$ may be determined for the new angle of incidence, and they are sufficient to characterize the two emerging beams (see Figure 1.30).

However, if the optical system is in any way sensitive to polarization, even when the incident light is unpolarized, it is important to recognize that the beam splitter can transmit and reflect different amounts of the “s” and “p” polarization states, as shown in Figure 1.31.

The amount of light output in each polarization state can be determined by simply breaking up the incident light into its two polarization components (s and p) and then calculating how much of each intensity is transmitted and reflected. For systems based on incoherent light, this level of detail is usually sufficient to keep track of the impacts of components such as optical filters on polarization.

For some optical systems – particularly those based on coherent light and that utilize or are sensitive to interference effects – the complete state of polarization should be tracked at every point through the system. In that case, it is important to understand that optical filters based on multilayer thin-film coatings not only reflect and transmit different amounts of intensity for the “s” and “p” polarization

Figure 1.31 The fraction of intensity reflection (I_R) and transmission (I_T) in terms of “s” and “p” polarization states [4]. Source: Courtesy of Semrock.



states but also impart different phases to the two different states. Both the amplitude and phase contributions can depend strongly on the wavelength of light. Thus, in general, an optical filter can act like a combination of a partial polarizer and a birefringent wave plate, for both reflected and transmitted light.

To determine the effect of an optical filter on the light in such a system, the incident light should first be broken up into the two fundamental components associated with the plane of incidence of the filter (s and p components). Then, the amplitude and phase responses of the filter for the “s” and “p” components should be applied separately to each of the incident light components to determine the amplitudes and phases of the reflected and transmitted light components. Finally, the reflected “s” and “p” components can be recombined to determine the total reflected light and its state of polarization, and likewise for the transmitted light. These steps are illustrated in Figure 1.32.

Because the polarization response of a tilted multilayer thin-film coating can be very strong, optical filters can make excellent polarizers. For example, a basic edge filter at a high angle of incidence exhibits “edge splitting” – the edge wavelength for light at normal incidence shifts to a different wavelength for p-polarized light than it does for s-polarized light. As a result, there is a range of wavelengths

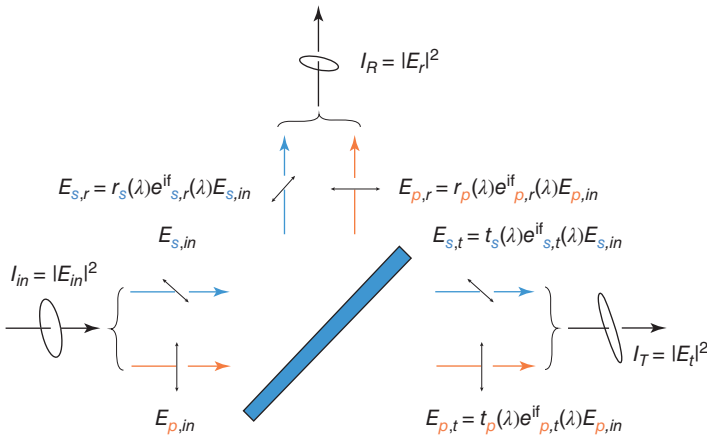


Figure 1.32 Distribution of I_R and I_T in terms of E_s and E_p of the incident light [4]. Source: Courtesy of Semrock.

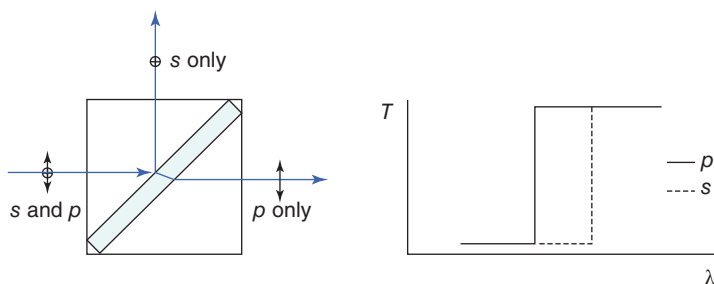


Figure 1.33 Thin-film plate polarizer [4]. Source: Courtesy of Semrock.

for which p-polarized light is highly transmitted while s-polarized light is highly reflected, as shown below (see Figure 1.33).

It is also possible to take advantage of an appreciable difference in reflected or transmitted phase for p- and s-polarized light over a region of the spectrum where the reflected and transmitted intensities are essentially equal, thus forming a wave plate.

1.9 Light Polarizing Devices

This section is a brief description of a number of devices that are used to modify the state of polarization of light. Their respective features are given below.

1.9.1 Polarizing Plate

A polarizing plate is a piece of film by itself or a film being held between two plates of glass. Adding salient iodine to preferentially oriented macromolecules will allow this film to have dichroism. Dichroism is a phenomenon in which discrepancies in absorption occur due to the vibration in the direction of incident light polarization. Since the polarizing plate absorbs the light oscillating in the arranged direction of the macromolecule, the transmitted light rays become linearly polarized. Despite its drawbacks of (i) limited usable wavelength band (visible to near infrared light) and (ii) susceptibility to heat, the polarizing plate is inexpensive and easy to enlarge.

1.9.2 Polarizing Prism

When natural light enters a crystal having double refraction the light splits into two separate linearly polarized beams. By intercepting one of these, linearly polarized light can be obtained; this kind of polarizing device is called a polarizing prism, and among those we find Glan–Thompson prism (Figure 1.34a) and Nicol prism (Figure 1.34b). A polarizing prism has higher transmittance than a polarizing plate and provides high polarization characteristics that cover a wide wavelength band. However, its angle of incidence is limited and it is expensive. In addition, when used in a polarizing microscope, this prism takes up more space than a polarizing plate and may cause image deterioration when placed in an

Figure 1.34 Polarizing prism.
 (a) Glan–Thompson prism
 (b) Nicol prism [5]. Source:
 Courtesy of Olympus.

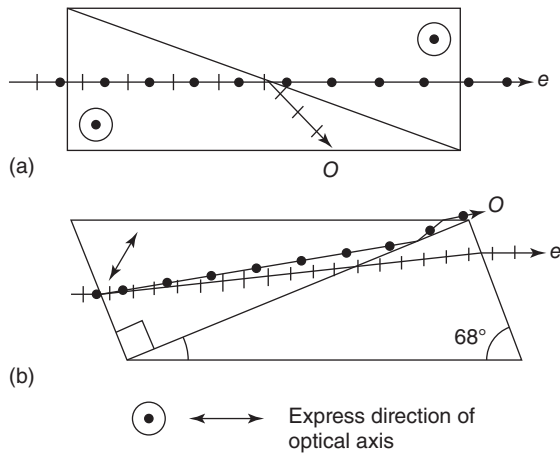


image forming optical system. For these reasons, a polarizing plate is generally used except when brightness or high polarization is required.

1.9.3 Phase Plate

A phase plate is used in the conversion of linearly polarized light and circularly polarized light, and in the conversion of the vibration direction of linearly polarized light. A phase plate is an anisotropic crystal that generates a certain fixed amount of retardation, and based on that amount, several types of phase plates (tint plate, quarter-wave plate, and half-wave plate) are made. When using a quarter-wave plate, a diagonally positioned optical axis direction can convert incident linearly polarized light into circularly polarized light and vice versa (Figure 1.35).

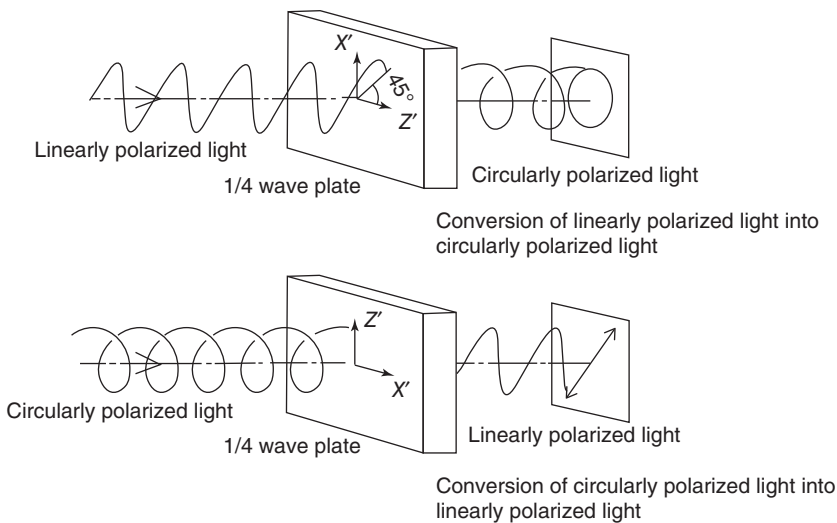


Figure 1.35 Quarter-wave plate conversion of linearly polarized light into circularly polarized light [5]. Source: Courtesy of Olympus.

A half-wave plate is mainly used for changing the vibration direction of linearly polarized light, and for reversing the rotating direction of circularly polarized and elliptically polarized light. Quarter-wave plates, half-wave plates, and tint plates are usually thin pieces of mica or crystal sandwiched in between the glass.

1.9.4 Optical Isolator

The optical isolator is a combination of a linearly PBS and a quartz quarter-wave plate as shown in Figure 1.36. Incident light is linearly polarized by the polarizer and converted to circular polarization by the quarter-wave plate. If any portion of the emerging beam is reflected back into the isolator, the quarter-wave plate produces a beam that is linearly polarized perpendicular to the input beam. This beam is blocked by the linear polarizer and not returned to the input side of the system. Two types of optical isolator are offered: a monochromatic optical isolator and a broadband optical isolator.

1.9.5 Optical Attenuators

An optical attenuator is built by combining two linear polarizers and a half-wave plate. The input and output polarizers are crossed so that no light passes through them; however, inserting the half-wave plate allows light to pass through the device. The amount of light is determined by the angle between the optical axis of the incoming polarizer and the half-wave plate. Placing the half-wave plate's optical axis at 45° to the incoming polarizer achieves maximum transmission; aligning the optical axis of the half-wave plate with either of the input or output polarizer optical axes gives the minimum transmission. How close the minimum is to zero transmission depends on the quality of the polarizer and the half-wave plate used in the device.

Replacing the half-wave plate with a liquid crystal (LC) variable retarder creates a variable attenuator. This configuration is shown in Figure 1.37. When we align the fast axis of the variable retarder at 45° to the input polarizer and modulate the retardance between half wave and full wave, transmission varies between the maximum and the minimum, creating an optical shutter chopper.

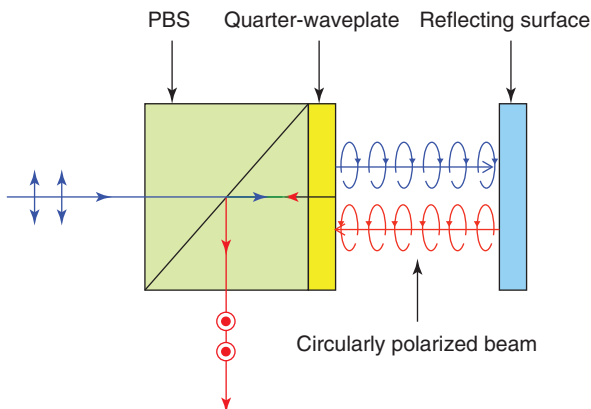


Figure 1.36 Demonstration of optical isolation [6]. Source: Courtesy of Union Optic.

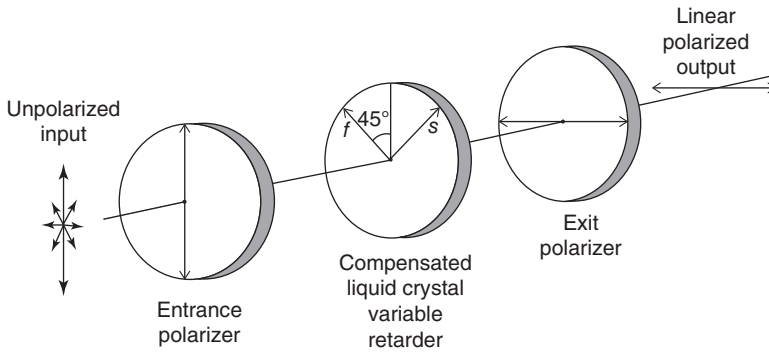


Figure 1.37 The variable attenuator configuration [7]. Source: Courtesy of Meadowlark Optics, Inc.

1.9.6 Polarization Rotator

A polarization rotator is an optical device that rotates the polarization axis of a linearly polarized light beam by an angle of choice. A simple polarization rotator consists of a half-wave plate in linear polarized light. Rotating the half-wave plate causes the polarization to rotate to twice the angle of the half-wave plate's fast axis with the polarization plane, as shown in Figure 1.38.

The polarization rotation induced by optically active crystals, such as wave plates, is reciprocal. If the polarization direction is rotated from right to left (say) on forward passage (as viewed by a fixed observer) it will be rotated from left to right on backward passage (as viewed by the same observer), so that back-reflection of light through an optically active crystal will result in light with zero final rotation, the two rotations having canceled out. This is best shown in the Figure 1.39.

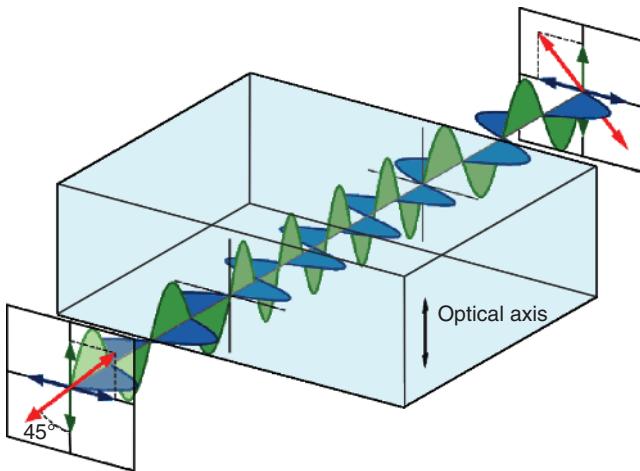


Figure 1.38 A half-wave plate rotates polarization by 90° [8].

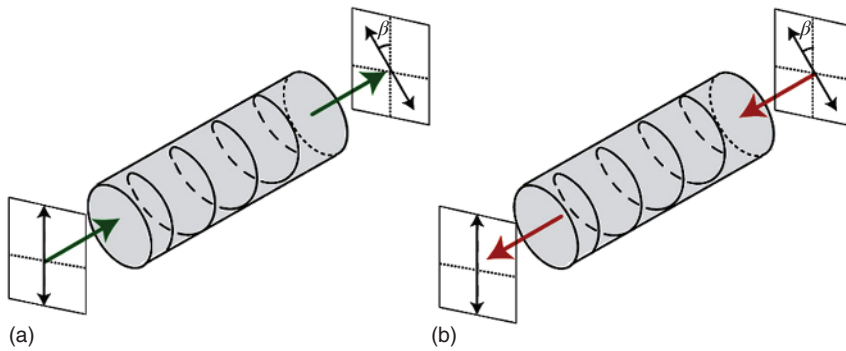


Figure 1.39 (a) Left to right polarization and (b) right to left polarization [9]. Source: Courtesy of Fosco.

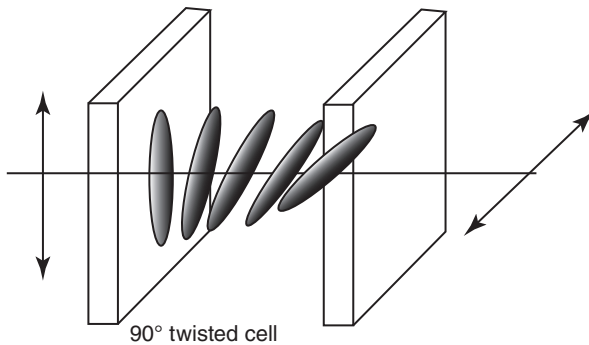


Figure 1.40 Liquid crystal twisted nematic polarization rotator cell [10]. Source: Courtesy of ARCOptix.

This is because the polarization rotation that occurs here is a result of a longitudinal spirality (represented as a helix) in the crystal structure. Hence, rotation following the handedness of the spiral in the forward direction will be opposed by the spiral in the backward direction.

Liquid crystal twisted nematic polarization rotator (TN cell) is very useful when one wants to rotate the orientation of linear polarization by a fixed amount, typically 45° or 90° . When light is traversing an LC twisted nematic cell its polarization follows the rotation of the molecules (Figure 1.40). The screen of any laptop computer is based on the same effect.

In optical systems, the polarization is often rotated by quartz retardation plates ($1/2$ or $1/4$ plates). Quartz plate shows high quality and good transmission performances especially in the UV region. However, such plates also present some disadvantages: They are expensive, function only for a narrow spectral bandwidth, and have a small incidence angle acceptance (field of view less than 2°). The liquid crystal nematic cells have therefore a large acceptance angle, function over a very large spectral range from Vis to NIR (if they are thick enough), and are less expensive. Optionally, by applying a voltage on the TN cell, the polarization rotation can be “switched off.” Also, when placing a 90° twisted cell between crossed polarizers, it can be used as a shutter.

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