

Contents

Preface — V

Acknowledgment — XI

1 Jordan algebras and symmetric cones — 1

- 1.1 Euclidean Jordan algebra — 1
- 1.2 The Peirce decomposition — 3
- 1.3 Symmetric cones — 8
 - 1.3.1 Symmetric cone associated with a Jordan algebra — 8
 - 1.3.2 Classification of the irreducible symmetric cones — 11
- 1.4 Groups of linear automorphisms — 13
 - 1.4.1 The group G — 13
 - 1.4.2 The orthogonal group — 14
 - 1.4.3 Frobenius transformation — 16
 - 1.4.4 The triangular group — 24

2 Generalized power — 31

- 2.1 Definitions and derivatives — 31
- 2.2 Properties — 34
- 2.3 Division algorithm — 50
- 2.4 Spherical polynomials — 53
 - 2.4.1 The gamma function and the generalized Pochhammer symbol — 53
 - 2.4.2 Spherical polynomials — 55
- 2.5 Spherical Fourier transform and Mellin transform — 56
 - 2.5.1 Spherical Fourier transform — 56
 - 2.5.2 Mellin transform — 57
 - 2.5.3 Multiplicative convolution — 59
- 2.6 Decomposition into G -orbits — 62
 - 2.6.1 Decomposition and boundary structure of a symmetric cone — 62
 - 2.6.2 Decomposition of the algebra into G -orbits — 62

3 Riesz probability distributions — 67

- 3.1 Gaussian probability distribution on a Euclidean space — 67
- 3.2 Lévy measure — 69
- 3.3 Generalized Riesz integrals — 69
- 3.4 Riesz measures — 69
 - 3.4.1 Definition of a Riesz measure — 69
 - 3.4.2 Description of the Riesz measures — 71
- 3.5 Riesz probability distributions — 79
- 3.6 Effect of the orthogonal group on a Riesz probability distribution — 80

3.7	Distributions of the Peirce components of a Riesz random variable — 82
3.7.1	Distributions of the Peirce components — 82
3.7.2	Distributions of the entries in the Cholesky decomposition of a Riesz random variable — 85
3.8	Connection between the Riesz probability distribution and the Gaussian — 86
4	Riesz natural exponential families — 93
4.1	Generalities on exponential families — 93
4.2	Natural exponential family generated by a Riesz measure — 96
4.2.1	Condition on the parameter s — 96
4.2.2	Variance function of a Riesz exponential family — 98
4.3	Characterization of a Riesz exponential families by a property of invariance — 103
4.4	Classification of the Riesz natural exponential families — 113
4.4.1	Construction of groups of linear transformations on a Jordan algebra — 114
4.4.2	Classification of the Riesz natural exponential families — 118
5	Tweedie scale — 125
5.1	Stable distributions — 125
5.2	The Tweedie scale — 134
5.3	The Riesz probability distribution as a Lévy measure — 141
6	Moments and constancy of regression — 143
6.1	Moments of a Riesz probability distribution — 143
6.1.1	Mean and variance of a Riesz probability distribution — 143
6.1.2	Mellin transform of a Riesz probability distribution — 143
6.2	Constancy of regression on the mean — 149
6.2.1	Definition and characterization — 149
6.2.2	Constant regression property for the Riesz probability distribution — 151
6.2.3	Constant regression of a scalar statistic — 154
7	Beta Riesz probability distributions — 159
7.1	Beta and hypergeometric functions — 159
7.1.1	Beta function — 159
7.1.2	Hypergeometric functions — 160
7.2	Beta-Riesz probability distributions — 164
7.3	Projection of a beta-Riesz probability distribution — 167
7.4	The expectation of a beta-Riesz probability distribution — 176

7.5	Characterization of the Riesz probability distribution without invariance — 182
7.5.1	Functional equations — 184
7.5.2	Characterization without invariance of the quotient — 187
7.6	Beta-Riesz probability distribution of the second kind — 189
8	Beta-Wishart distributions — 193
8.1	A characterization of the beta-Wishart probability distribution — 197
8.2	Product of beta-Wishart random variables — 199
8.3	Stability property — 201
8.4	Characterizations by constancy regression — 204
9	Beta-hypergeometric distributions — 213
9.1	Continued fractions in a symmetric cone — 213
9.1.1	Definition and properties — 213
9.1.2	Convergence — 216
9.2	Beta-Gauss hypergeometric distributions — 228
9.2.1	Definition — 228
9.2.2	Characterizations of the beta-Gauss hypergeometric distributions — 229
9.3	Generalized beta distribution — 234
10	Riesz-Dirichlet distributions — 239
10.1	Definition — 239
10.2	Projections of a Riesz-Dirichlet distribution — 242
10.3	Wishart-Dirichlet distribution — 246
10.3.1	Some results involving determinants — 246
10.3.2	Product and independence properties — 249
10.3.3	Mauldon characterization of the Wishart-Dirichlet distribution — 253
10.3.4	A distributional property — 256
10.4	A generalization of the Dirichlet distribution — 258
11	Riesz inverse Gaussian distribution — 261
11.1	Modified generalized Bessel function — 262
11.2	Connection with the Riesz probability distribution — 264

Bibliography — 267

Index — 273

Index of notations — 275